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THEORETICAL ANALYSIS OF FOUR-BAR MECHANISMS

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THEORETICAL ANALYSIS OF FOUR-BAR MECHANISMS

Introduction

Four-bar linkages possess several advantages as mechanisms that can generate straight-line motion. Probably the first engineering application of a straight-line mechanism was made by James Watt in constraining the motion of the piston rod of his beam engine along a straight line path. The straight line generated by all such mechanisms is, however, only approximate though quite close. Theoretically the line is exactly straight for only an infinitesimal distance. Beyond this range it begins to deviate; the amount of the deviation is certainly of great importance to the designer who intends to incorporate such a linkage in his design. Thus, finding the quantitative answer to the problem is a most necessary part of this investigation.

The production of straight-line motion by four-bar linkages has been the subject of investigation of many distinguished kinematicians. Chebishev, Burmester, Mueller, Allievi and other past researchers as well as some more recent investigators like deJonge, Meyer zur Capellan, Bottema, Veldkamp, Wunderlich, Beyer, Tesar, Freudenstein and others have done much important work. Some 453 references dealing with basic principles, general properties and special details of straight-line linkages have been uncovered in the technical literature. All of these contain mostly qualitative information; some analytical and graphical design procedures for the general solution of approximate straight-line motion have been developed, mostly by the first group of researchers. No one has, however, provided procedures that would permit the designer to select a four-bar linkage with a required length and accuracy of straight-line output. This is specifically the purpose and accomplishment of this investigation.

Terminology

The analytical foundation for the investigation of straight-line linkages was developed by Delbert Tesar in the work that he performed for his Doctoral dissertation (1). Formulation concerning the basic concepts of curvature theory and that of finitely seperated positions of the moving plane is involved.

Curvature theory as applied in the investigation is based largely upon the idea of order of contact between algebraic curves. This is closely associated with the number of infinitesimally separated points common to the two contacting curves. As the two intersections of a secant to a curve approach each other, the secant assumes in the limit the tangent position in which the two intersections become two infinitesimally separated points. This constitutes first order contact. It should be noted that the tangent is a fair approximation to the curve in the region of contact. Second-order contact exists when three infinitesimally separated points (or two infinitesimally separated tangents) are common to the two curves. The idea can be extended indefinitely and used to include properties of coupler curves compatible with the concept.

The Ball and Burmester points have such properties. Involved in the location of these points are the inflection circle and the cubic of stationary curvature. The inflection circle is the locus of those points in the moving plane that are momentarily inflections of their point paths, that is, have an infinite radius of curvature. The circle is defined in polar coordinates by the equation

$$r = PJ \sin \alpha = PD \qquad (1)$$

where the symbols are as in Fig. 1. The Ball point is of third-order contact.

The cubic of stationary curvature is the locus of points in the moving plane that are momentarily passing through path elements which have a zero first derivative of the radius of curvature with respect to distance along the path. The polar equation of the cubic is (see Fig. 2).

$$\frac{1}{r} = \frac{1}{M \sin \alpha} + \frac{1}{N \sin \alpha} \tag{2}$$

where $\underline{M}$ and $\underline{N}$ are constants found using two known cubic points such as the pin joints in a four-bar linkage.

The intersection of the inflection circle with the cubic of stationary curvature is a Ball point. It can be shown to involve four infinitesimally separated positions on a straight line, which makes it of third order contact. The Ball point is, therefore, a point on the body, which traces a path having an infinite radius of curvature and the rate of change of the radius of curvature is momemtarily zero. It thus traces an approximate straight line path over a limited range of motion.

There are some points on a body moving with general coplanar motion which describe, momentarily, path elements having both the first and second derivatives of the radius of curvature with respect to distance along their point paths equal to zero. These are called Burmester points which have fourth-order contact with their osculatory circles, that is, five infinitesimally separated positions of a Burmester point lie on a circular arc. If the Burmester point lies on the inflection circle coincident with Ball's point, five infinitesimally separated positions of this Ball-Burmester point will lie on a straight line, probably yielding a very satisfactory approximate straight-line mechanism.

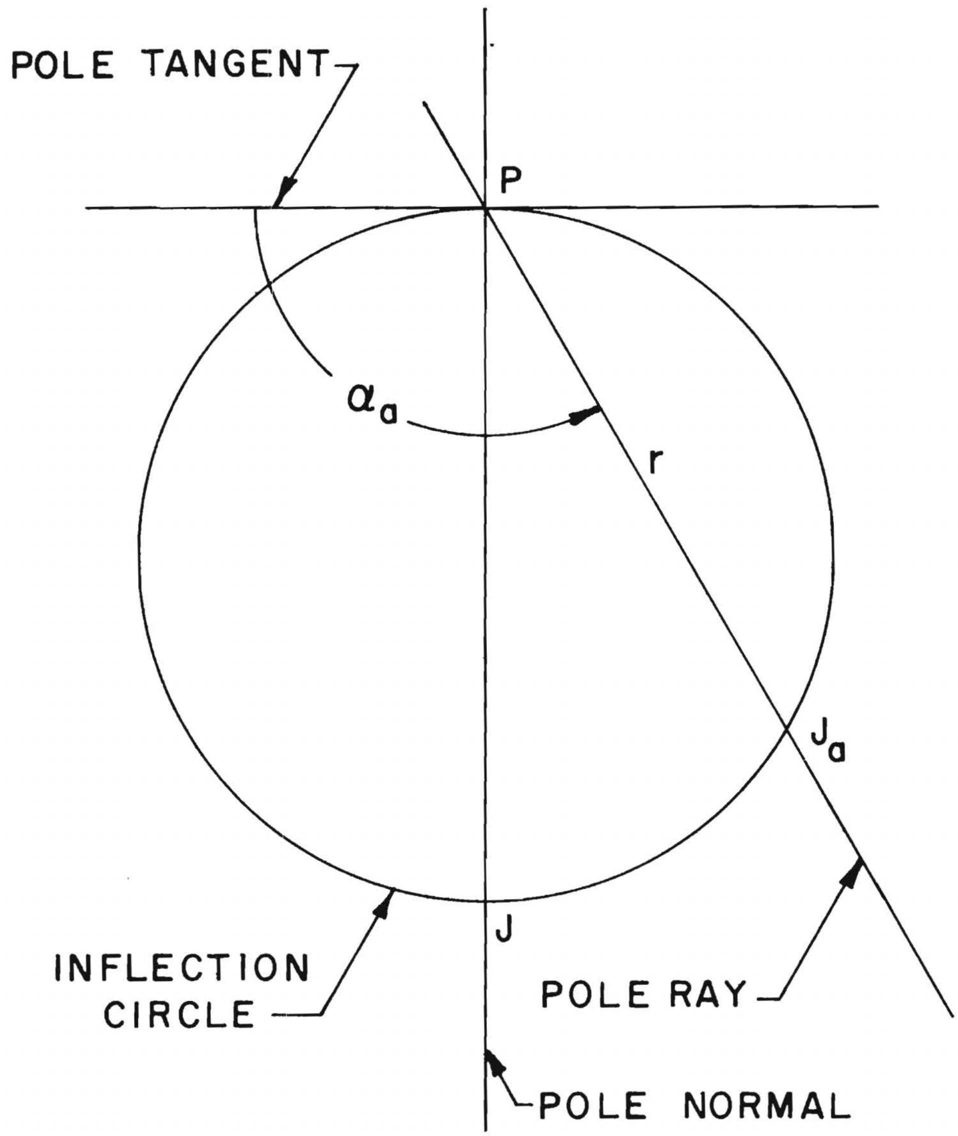


Figure 1. The Inflection Circle.

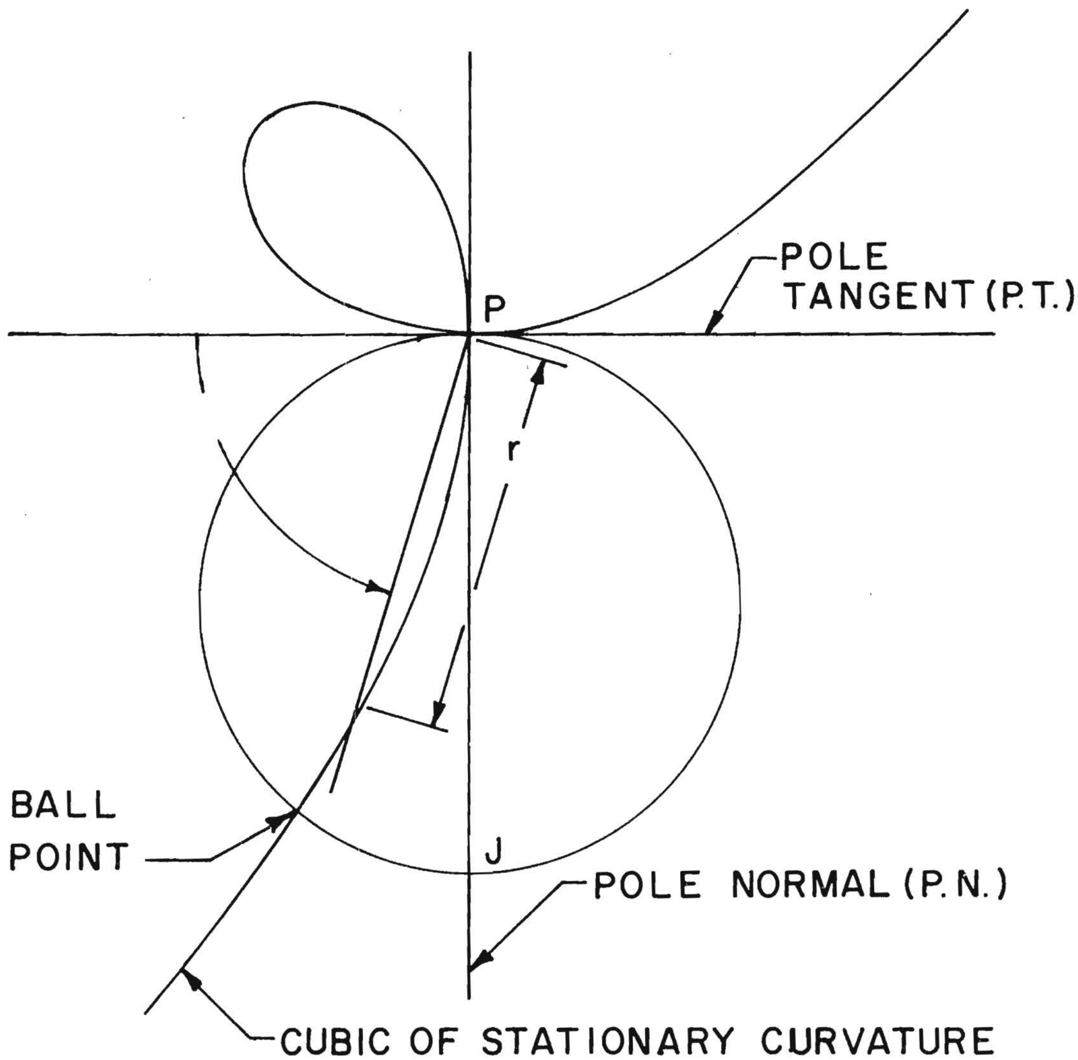


Figure 2. Cubic of Stationary Curvature.

When a Burmester point coincides with the Ball point, the remaining three Burmester points (there are four Burmester points for every position of the moving plane of a four-bar linkage) are collinear. It is thus possible for a second Burmester point to coincide with the Ball point. In such a situation the Ball-double Burmester point lies on the coupler center line of a four-bar linkage formed by using the two remaining Burmester pairs (a point pair is the coupler point and its corresponding center which can be found using the Euler-Savary relation) as rigid links. The design of the straight-line mechanism is then simplified because the number of independent parameters is reduced from three to two.

The possible selection of straight-line four-bar linkages is enhanced by the concept of alternate linkages. Each Burmester pair, constituting a pin joint-fixed pivot, is actually a crank. Therefore, if the pins of any two pairs are joined, a four-bar linkage is formed. Thus, if all four Burmester points are real, six different combinations or alternate linkages become possible. Each will allow the primary cranks to give the same motion to the coupler link for the five infinitesimally separated positions. Publication (1) should be studied for a detailed presentation of curvature theory, its formulation, and application to approximate straight-line four-bars.

The formulation of the theory of finitely separated positions of the moving plane in coplanar motion is rather extensive. Space will not be taken here to detail it because it is also contained in Publication (1). Based on the fundamentals propounded by Burmester and Bottema, original derivations of the following, needed in this study, are to be found in the referenced dissertation.

1. The equation of the cubic circle point curve which is the locus of all points in the moving plane that pass through four finitely

separated positions on a circular arc.

2. The quartic equation for the general case of five finitely separated positions of a point of the moving plane on a circular arc.
3. The quadratic equation for the unknown Burmester points of a given four-bar linkage which is specified in five finitely separated positions.
4. The cubic equation for the three unknown Burmester points where the fourth Burmester point is prescribed in five finitely separated positions on a straight line.
5. The quadratic equation for the unknown Burmester points of a given slider-crank mechanism which is specified in five finitely separated positions.

Analysis

Above principles are used in multiple position design of approximate straight-line four-bar linkages. The proper combination of curvature and finitely separated position theory eliminate in some cases the difficulty of having to select satisfactorily three independent design parameters by reducing the requirement to only two. It also increases both the length and accuracy of the approximate straight-line segments generated by the linkages.

Formulation is also developed for the design of cases in which various combinations of contacts between the coupler curve and a straight line are considered. Cases such as one prescribed intersection with a Ball point, one prescribed tangent with a Ball point and one prescribed intersection with a Ball-Burmester point are worked out analytically. Unfortunately these special cases proved to present programming problems which were not debugged during the course of this investigation. Design charts were therefore, not prepared, for these as for the other cases.

As implied, data needed to construct the design charts, constituting the most useful output of the investigation, were obtained using digital computers, Burroughs 220 and/or 5500. The primary information is the length of straight line for a given accuracy. Other design information, however, includes transmission angles, crank rotation angles and longest-to-shortest link ratios for, at least, some of the cases investigated.

Cases Studied

The classical mechanisms of Watts, Evans and the symmetric types, such as that of Roberts and Chebishev, were thoroughly investigated. Since the proportions of the mechanisms are already known, the basic computer program readily provided the design chart data.

Third and fourth order contact cases were also thoroughly investigated. Use of the Ball point, the Ball-Burmester, the Ball-double Burmester and the Ball-Burmester at the inflection pole was made, these cases were analyzed, design data obtained and plotted.

Fifth order contact was studied as well. This case, however, produced only a small number of practical linkages. The mechanisms found are, therefore, included as "six point" linkages on the Ball-Burmester charts.

The precision point and tangent combinations did not work out as the analytics promised. A set of charts was, however, prepared that can be used, along with the Ball-Burmester charts, to select mechamisms of the precision and intersection points type.

The computer formulation developed by Freudenstein and Sandor for five finitely-separated precision points was used; the five precision points are taken on a straight line. The infinite number of precision-point combinations possible was reduced by considering only equal spacing of points and the Chebishev distribution (projections on the diameter of points equally spaced on a circular arc). The coupler path did pass through the precision points, but deviations from the straight line in between the precision points were excessive and irregular. The approach had to be discarded as inadequate for this purpose.

Publications

Since it seems unwise to detail the investigation completely in this report, the publications resulting therefrom are listed and assumed a part of the report.

(1) Tesar, D., The Analytical Theory of Coplanar Motion Applied to Approximate Four-Bar Straight-Line Mechanisms, Doctor's Dissertation, Georgia Institute of Technology, 1964, 177 pages.

(2) Hiegel, J. E., Design of Classical Straight-Line Mechanisms, Master's thesis, Georgia Institute of Technology, 1965, 87 pages.

(3) Tesar, D. and J. P. Vidosic, Analysis of Approximate Four-Bar Straight-Line Mechanisms, Journal of Engineering for Industry, August 1965, pp 291-297.

(4) Vidosic, J. P. and D. Tesar, The Selection of Watt's Four-Bar Straight-Line Mechanisms, presented at ASME Mechanisms Conference, October 10-12, 1966, Purdue University.

(5) Vidosic, J. P. and D. Tesar, Selection of Four-Bar Mechanisms Having a Required Approximate Straight-Line Output, Part I - The General Ball-Burmester Point Mechanism, selected for publication in Journal of Mechanisms, Pergamon Press.

(6) Tesar, D. and J. P. Vidosic, Part II - Ball-Burmester Point at the Inflection Circle Mechanism, accepted for publication in Journal of Mechanisms, Pergamon Press.

(7) Vidosic J. P. and D. Tesar, Part III - Ball-double Burmester Point Mechanisms, selected for publication in Journal of Mechanisms, Pergamon Press.

(8) Tesar, D., and J. P. Vidosic, The Analytical Theory of Finitely Separated Positions of the Moving Plane in Coplanar Motion, paper written but not offered for presentation or publication to date.

(9) Two additional papers are planned; one dealing with the Evans and the other with the symmetric linkages.

Design Data

The most useful result of the investigation is the accumulated computer data which can be used in the design of straight-line mechanisms. The usefulness of such information depends largely upon its ease of application. It must also be readily available. The first requirement is believed to have been well met; as will be explained; the charts can be readily used. The second item is another matter. Much of the information is contained in some of the published articles, but not all of it. And, how readily available these publications become is beyond the author's control. Extra copies of this report will be printed so it can be offered upon request.

The data has been reduced into a series of charts which can be used, easily, by a designer to select a four-bar linkage, providing a desired straight line of specified accuracy. Accuracy is defined as the deviation of the coupler path from the exact straight line. Deviations of 0.01 to 0.0001 unit have been used in the investigation. Thus the path extends in both directions from the design position until it just begins to exceed the specified deviation. The distance between these two end points is the approximate straight line.

In order that a basis of comparison and evaluation of the numerous mechanisms be made possible, a size index is developed. It has taken the very satisfactory form of a "unit length" or "unit number" defined

$$UN = \frac{Q + R + S + T + (M + N)/2}{5} \qquad (3)$$

where the letters are the link lengths indicated in Fig. 3. All length and deviation computer data are adjusted to the index; linkages are thus reduced to the same relative size. Chart readings in turn become an indication of the straight line output relative to an overall size of unity for the linkage.

When used, chart readings have to be adjusted by way of the UN index; chart link lengths are determined and the UN computed. The straight line length and its deviation read off the charts are then adjusted to obtain the actual straight length and its deviation in inches. It must be observed that if the chart mechanism is increased or decreased in size, the straight line length as well as its deviation change accordingly. For example, suppose

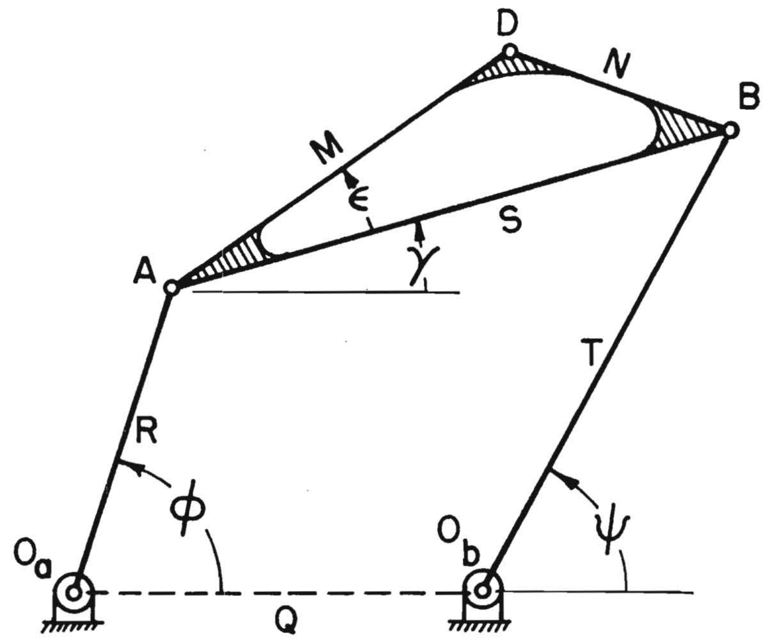


Figure 3. Linkage Symbols.

that the straight length is 0.75 inch and the deviation is 0.001 inch. Then by increasing the size of the mechanism by a factor of 10, the values would become 7.5 inches and 0.01 inch.

In order to remove from consideration disproportionate linkages, the computer was instructed to eliminate all mechanisms in which the largest length exceeds the smallest by more than a given number of times. Other limitations used were such items as minimum straight line length, range of crank length, crank rotation angle and coupler inclination. The selection is thus eased by the elimination of a large number of mechanisms that would prove awkward and impractical. Some variation in the kind of data and the form of its presentation is to be found depending upon the general group of linkages. The specific manipulation of charts can vary some as well. As each set of design charts is presented a brief explanation and example problems are given to specifically illustrate the proper use of the charts.

The General Ball-Burmester Point Case

Mechanisms in this group are limited to cases where the ratio of longest to shortest link length does not exceed ten and the straight lines are at least 0.75 unit long within the specified accuracy. Maximum deviations of 0.01 and 0.0001 are used. Contained within the cross hatched areas are the crank-lever types of mechanisms, while the rest are double-cranks. The small number of "six point" or fifth-order contact linkages are also shown on these charts. In addition some explanatory notes are given as well.

Charts BB1 to BB14 represent linkages labeled alternates A,B (or linkage O_aABO_b). The contour lines for the ratio of longest to shortest link lengths are drawn as dotted lines and the ratio numbers inclosed in small rectangles. The solid contour lines mark loci of mechanisms producing the straight-line outputs of marked value and accuracy specified on the chart.

Charts BB15 to BB36 represent linkages labeled alternates B,C (or linkages O_bBCO_c). The crank rotation as well as the minimum transmission angle at the follower pin are also given.

The three parameters involved in the selection are α_a which is constant for any given chart (shown thereon), and α_a and α_b (or α_b and α_c) which constitute the chart coordinates.

Needed in addition to the equations already given are the following.

Pin point location -

$$PA = \frac{[(3W + 1) \tan \alpha_d + VW] \sin \alpha_a}{(W + 1) \tan \alpha_a + W (2 \tan \alpha_d + V)}$$

$$PB = \frac{[(3W + 1) \tan \alpha_d + VW] \sin \alpha_b}{(W + 1) \tan \alpha_b + W (2 \tan \alpha_d + V)} \qquad (4)$$

where $V = \tan \alpha_a + \tan \alpha_b$ and $W = (\tan \alpha_a) (\tan \alpha_b)$.

The fourth Burmester point C is determined by calculating the polar coordinates

$$\alpha_c = \arctan\left\{-\frac{2\tan\alpha_d + V}{W + 1}\right\}$$

$$PC = \frac{[(3W + 1)\tan\alpha_d + VW]\sin\alpha_c}{(W - 1)(2\tan\alpha_d + V)} \tag{5}$$

The centers of curvature of the point-paths of the Burmester points O_a, O_b, O_c are located by

$$PO_a = -\frac{PA\sin\alpha_a}{PA - \sin\alpha_a}$$

$$PO_b = -\frac{PB\sin\alpha_b}{PB - \sin\alpha_b} \tag{6}$$

$$PO_c = -\frac{PC\sin\alpha_c}{PC - \sin\alpha_c}$$

The particular linkage dimensions can now be computed as follows

$$O_aA = \frac{(PA)^2}{PA - \sin\alpha_a}\ ; \qquad O_bB = \frac{(PB)^2}{PB - \sin\alpha_b}$$

$$AB = \{(PA)^2 + (PB)^2 - 2(PA)(PB)\cos(\alpha_b - \alpha_a)\}^{\frac{1}{2}}$$

$$O_aO_b = \{(PO_a)^2 + (PO_b)^2 - 2(PO_a)(PO_b)\cos(\alpha_b - \alpha_a)\}^{\frac{1}{2}} \tag{7}$$

$$AD = \{(PA)^2 + (PD)^2 - 2(PA)(PD)\cos(\alpha_d - \alpha_a)\}^{\frac{1}{2}}$$

$$BD = \{(PB)^2 + (PD)^2 - 2(PB)(PD)\cos(\alpha_d - \alpha_b)\}^{\frac{1}{2}}$$

Example

Use of the charts and steps involved in designing the mechanisms are illustrated in the following example.

Suppose that a four-bar linkage is to be designed to generate an approximate straight line output of 1.5 units at a maximum deviation of 0.01 unit. It is also desirable to have a large crank rotation angle and a reasonable transmission angle.

An examination of the charts indicates that a suitable mechanism occurs in the B,C alternate group. Charts BB21-23 contain a linkage having an output of 1.6 units, a crank rotation of 180 degrees, and a minimum transmission angle of 30 degrees. The parameters read off the charts are

$$\alpha_d = 65° \qquad \alpha_a = 35° \qquad \alpha_b = 5°$$

The following list of preliminary calculations must be performed to determine the configuration of the linkage. The constants V and W are

$$V = \tan \alpha_a + \tan \alpha_b = 0.7002 + 0.0875 = 0.7877$$

$$W = \tan \alpha_a \times \tan \alpha_b = 0.7002 \times 0.0875 = 0.0613$$

The polar coordinates of pin joints A and B are determined by Eqs. (4) as

$$PA = \frac{[(3 \times 0.0613 + 1)\ 2.1445 + 0.7877 \times 0.0613]\ 0.5736}{(0.0613 + 1)\ 0.7002 + 0.0613\ (2 \times 2.1445 + 0.7877)} = 1.406$$

$$PB = \frac{[(3 \times 0.0613 + 1)\ 2.1445 + 0.7877 \times 0.0613]\ 0.0872}{(0.0613 + 1)\ 0.0875 + 0.0613\ (2 \times 2.1445 + 0.07877)} = 0.560$$

The polar coordinates of pin joint C are by Eqs. (5)

$$\alpha_c = \tan^{-1} \left\{ -\frac{2 \times 2.1445 + 0.7877}{0.0613 + 1} \right\} = \tan^{-1} (-4.77)$$

$= -78.2° = 101.8°$ (Note that the polar system depends upon directed pole rays; the angle should be calculated between 0 and 180° to eliminate errors of application). And

$$PC = \frac{[(3 \times 0.0613 + 1)(2.1445) + (0.7877)(0.0613)](0.9789)}{(0.0613 - 1)(2 \times 2.1445 + 0.7877)} = -0.531$$

The polar coordinates for the fixed pivots O_a, O_b and O_c are computed using Eq. (6) as

$$PO_a = - \frac{1.406 \times 0.5736}{1.406 - 0.5736} = -0.964$$

$$PO_b = - \frac{0.560 \times 0.0872}{0.560 - 0.0872} = -0.103$$

$$PO_c = - \frac{(-0.531)(0.9789)}{(-0.531) - (0.9789)} = 0.344$$

Eq. (1) yields the polar coordinate for output point D, remembering that the inflection circle diameter PJ is one unit

$$PD = 0.9063$$

The significant points should now be laid out graphically as in Fig. 4 (the inflection circle has a unit diameter of 3"). Since pin joints A, B and C are collinear, the calculations appear satisfactory.

If the linkage satisfies the designer's proportions, the following calculations will complete the determination of linkage dimensions. These are Eqs. (7) applied to alternate B,C.

$$O_bB = \frac{(0.560)^2}{0.560 - 0.0872} = 0.667$$

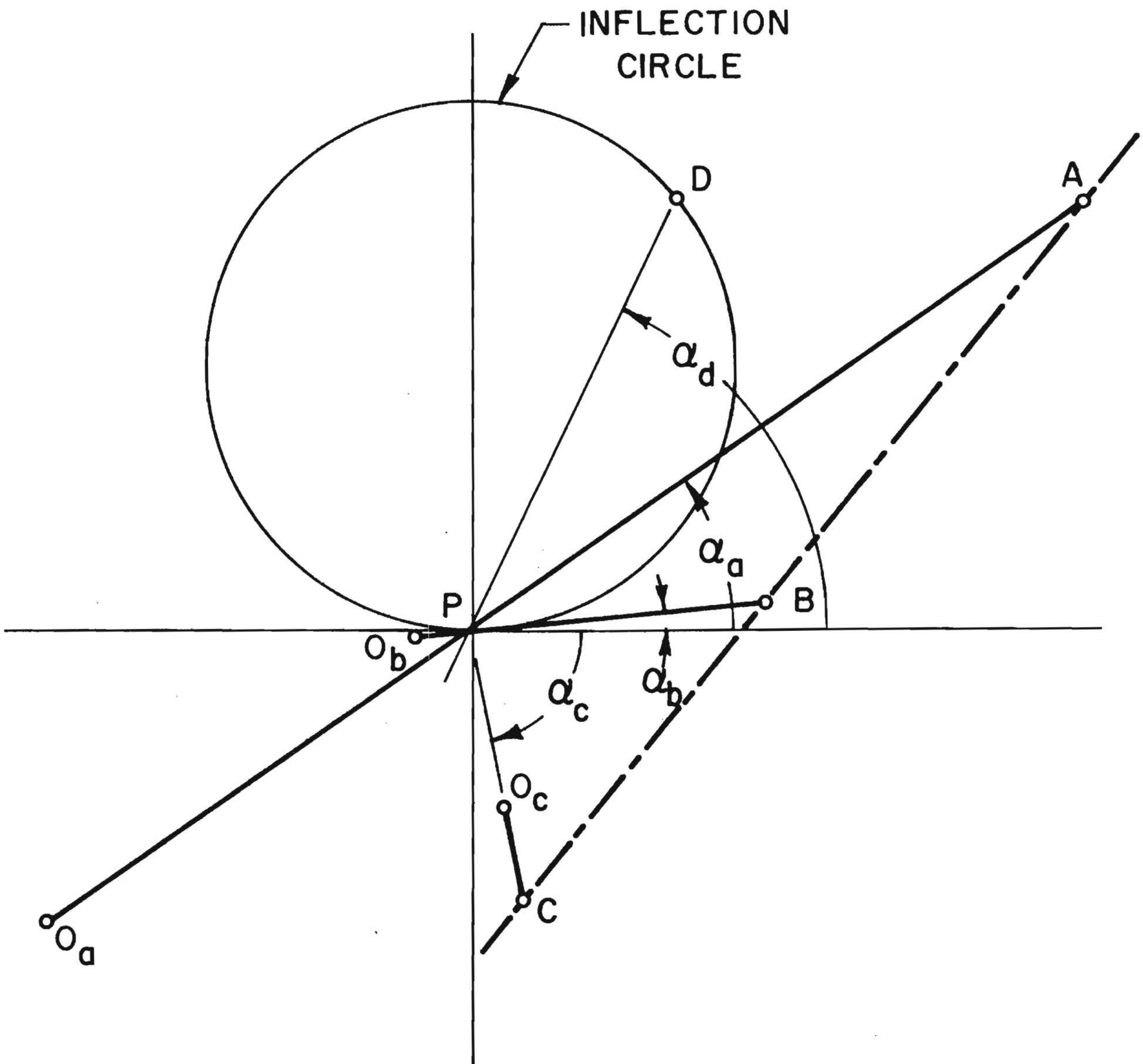


Figure 4. Pin Joint or Burmester Collinearity.

$$0_cC = \frac{(-0.531)^2}{(-0.531) - (0.9789)} = -0.188$$

$$0_b0_c = [(-0.103)^2 + (0.344)^2 - 2(-0.103)\ (0.344)\ (0.1184)]^{\frac{1}{2}} = 0.370$$

$$BC = [(0.560)^2 + (0.531)^2 - 2(0.560)\ (0.531)\ (0.1184)]^{\frac{1}{2}} = 0.725$$

$$BD = M = [(0.560)^2 + (0.9063)^2 - 2(0.560)\ (0.9063)\ (0.5000)]^{\frac{1}{2}} = 0.793$$

$$CD = N = [(0.531)^2 + (0.9063)^2 - 2(0.531)\ (0.9063)\ (-0.8007)]^{\frac{1}{2}} = 1.37$$

The size index of the linkage is calculated by

$$UN = \frac{Q + R + S + T + \frac{M + N}{2}}{5}$$

$$= \frac{0.370 + 0.667 + 0.725 + 0.188 + \frac{0.793 + 1.37}{2}}{5} = 0.606$$

If the linkage is to have a unit size index, all the preceding dimensions must be divided by the above value of UN. This would result in a linkage of the size recorded in the charts. Hence, the actual straight-line output is characterized by

$$L_a = (L_c)\ (UN) = (1.6)\ (0.606) = 0.96$$

$$D_a = (D_c)\ (UN) = (0.01)\ (0.606) = 0.006$$

Note that $L_a \approx (1.4)\ (0_bB)$ appears well satisfied by the coupler curve in Fig. 5. (Fig. 5 is approximately 1.4 times the Fig. 4 scale.)

Suppose it is now required that the shortest link 0_cC be made 2.0 inches long. The magnification factor is

$$M.F. = \frac{2.0}{0_cC} = \frac{2.0}{0.188} = 10.6$$

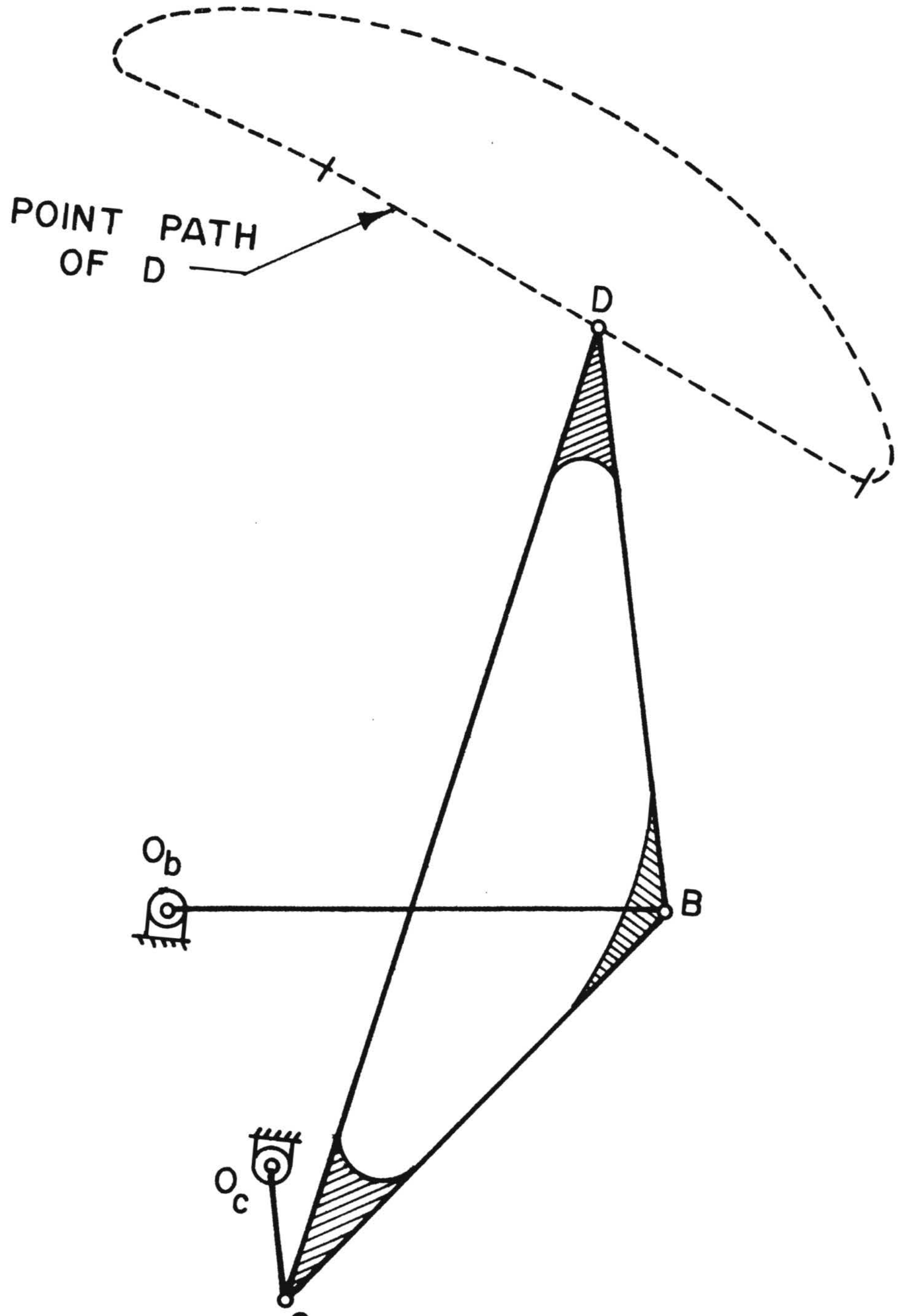


Figure 5. Example Ball-Burmester Linkage.

If the dimesnions calculated earlier are multiplied by this factor, the size index becomes

$$UN' = 10.6 \times 0.606 = 6.42$$

and the straight line output is characterized by the dimesions

$$L_a' = (UN')\ (L_c) = (6.42)\ (1.6) = 10.24''$$

$$D_a' = (UN')\ (D_c) = (6.42)\ (0.01) = 0.0642''$$

Considering Chart BB24, note that $L_c \approx 0.57$ and $D_c = 0.0001$. Hence a smaller range of the coupler curve produces a straight-line output characterized by

$$L_a' = (UN')\ (L_c) = (6.42)\ (0.57) = 3.64''$$

$$D_a' = (UN')\ (D_c) = (6.42)\ (0.0001) = 0.00064''$$

Although this portion is approximately 0.4 times the larger segment, it is 100 times more accurate. As indicated in previous articles, this disproportionate increase in accuracy is characteristic of mechanisms based on curvature theory.

BALL-BURMESTER POINT

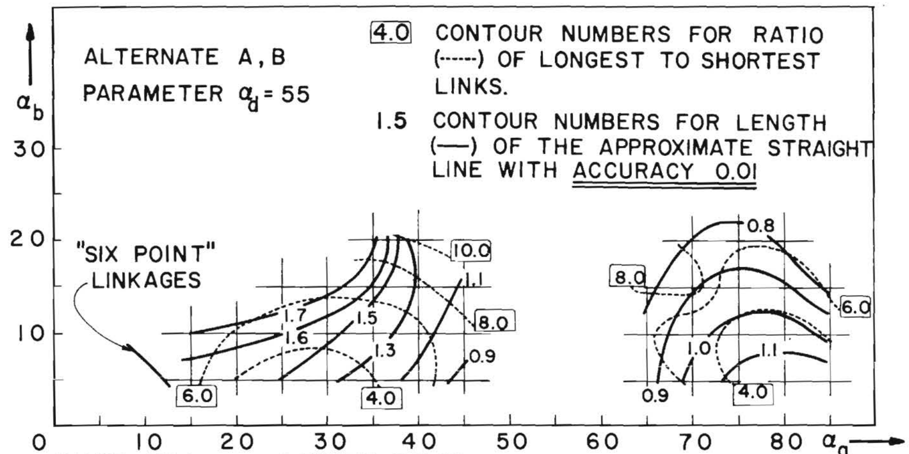


CHART BB 1 LENGTH-RATIO

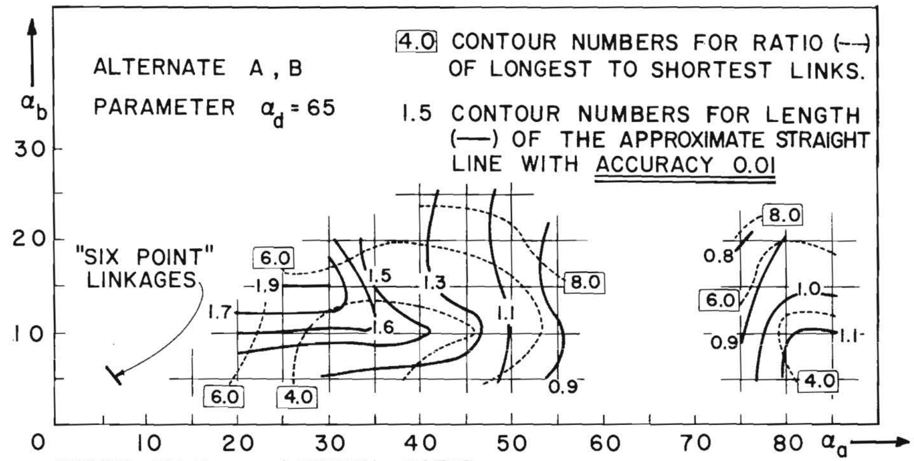


CHART BB 3 LENGTH-RATIO

23

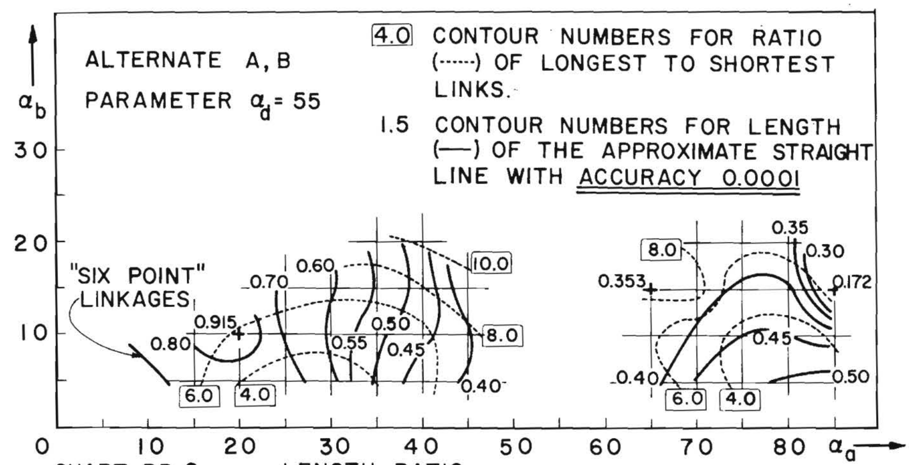


CHART BB 2 LENGTH-RATIO

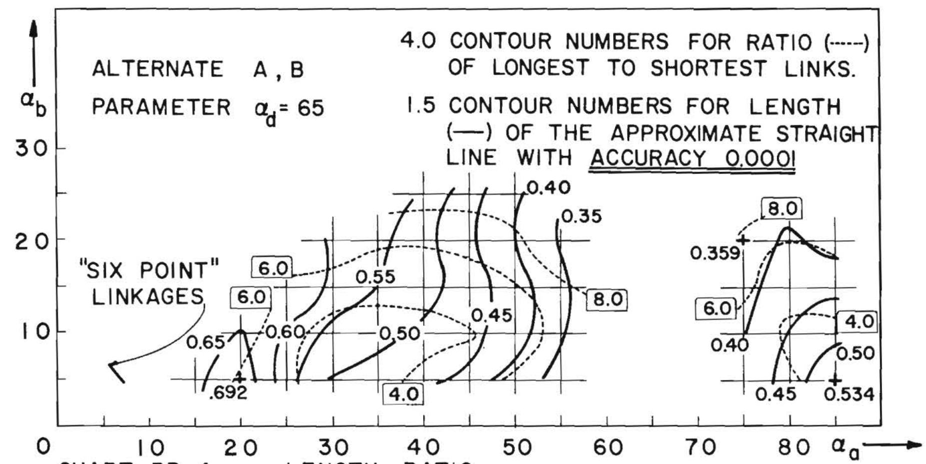


CHART BB 4 LENGTH-RATIO

BALL-BURMESTER POINT

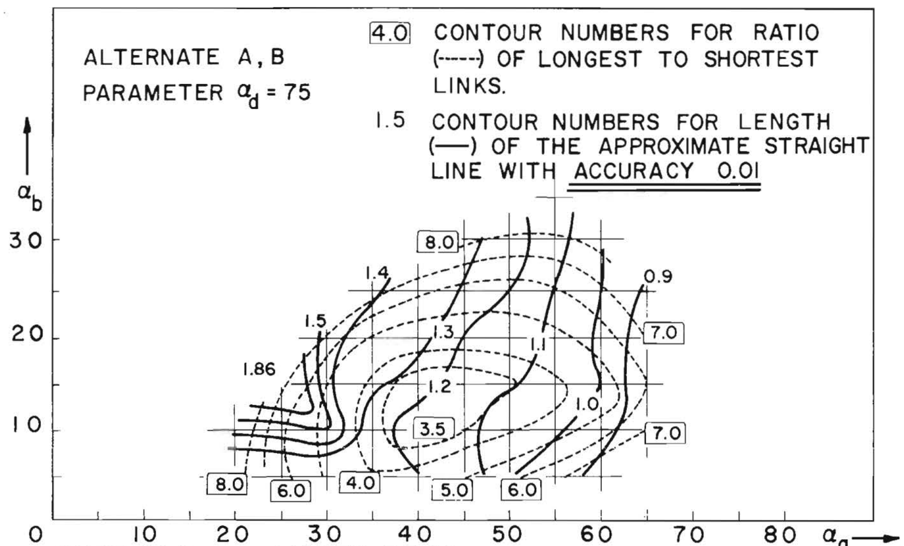


CHART BB 5 LENGTH-RATIO

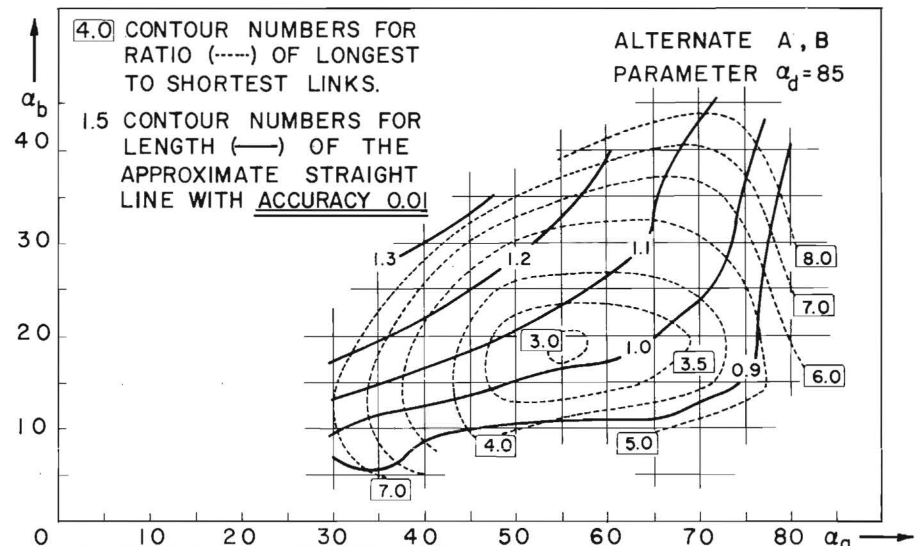


CHART BB 7 LENGTH-RATIO

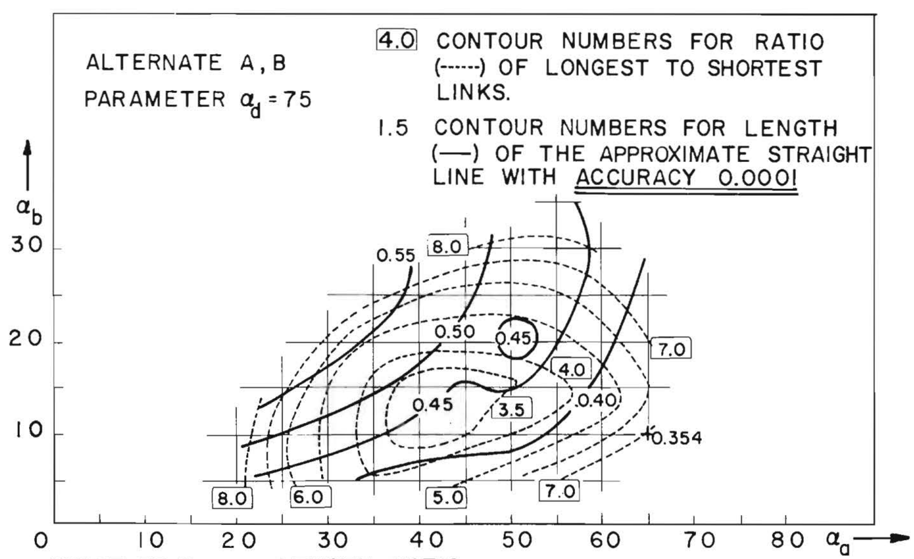


CHART BB 6 LENGTH-RATIO

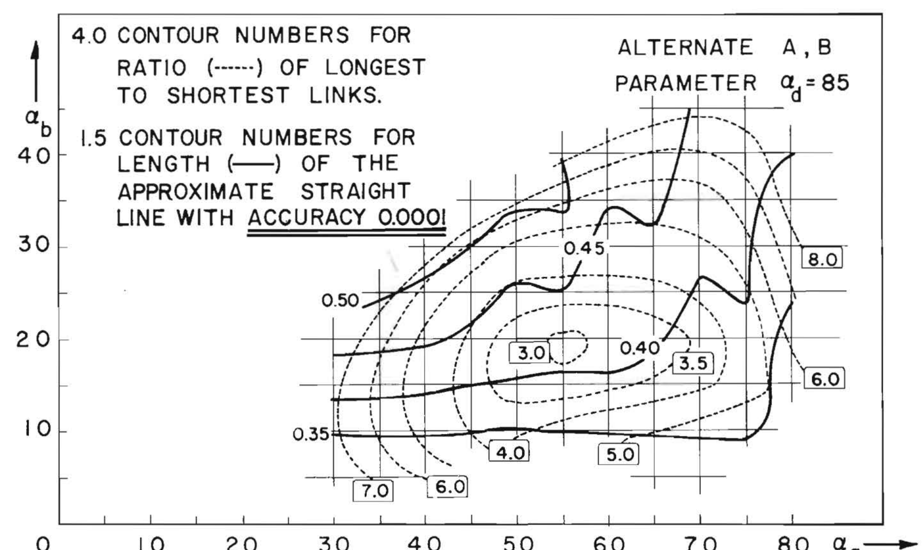


CHART BB 8 LENGTH-RATIO

BALL-BURMESTER POINT

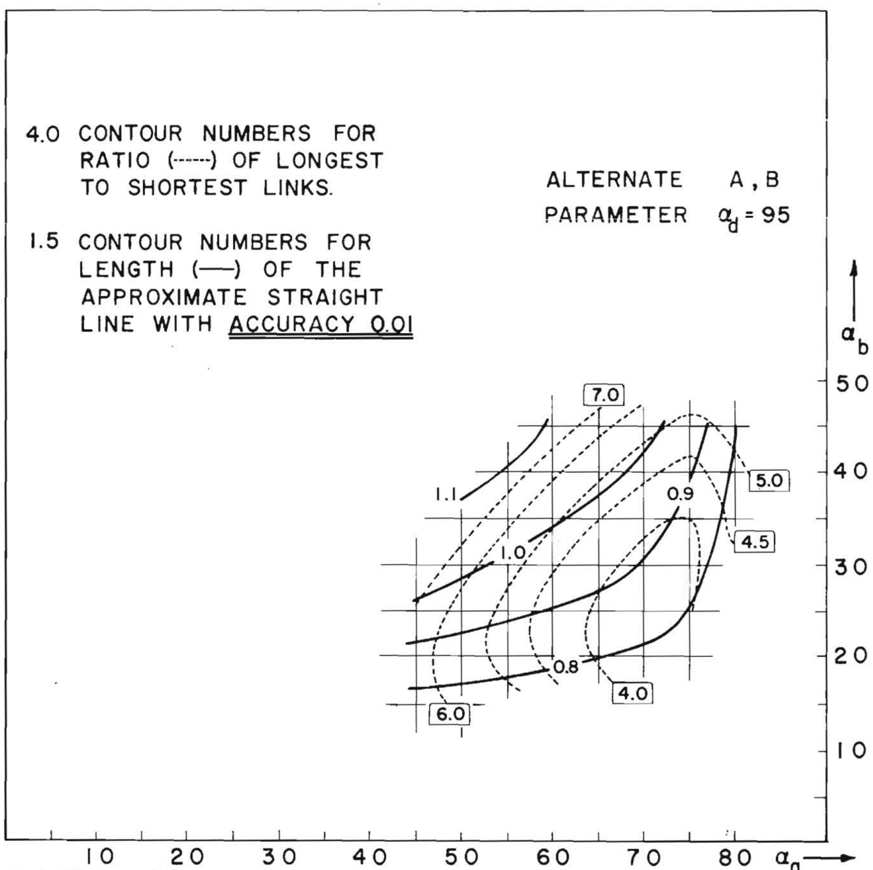


CHART BB 9 LENGTH-RATIO

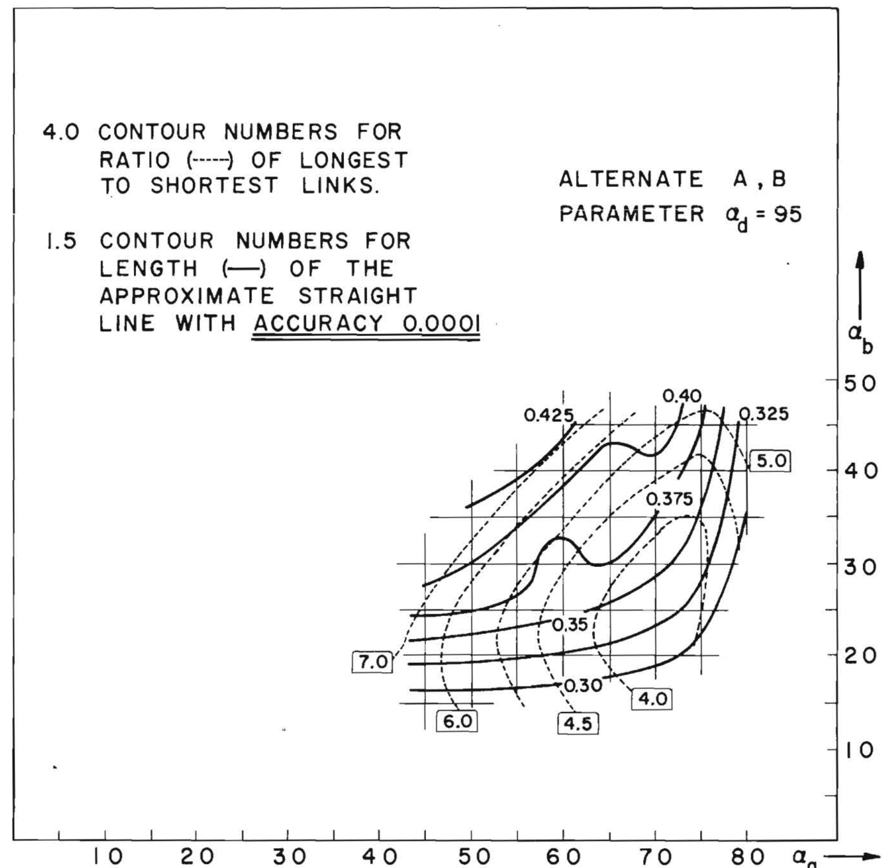


CHART BB 10 LENGTH-RATIO

BALL-BURMESTER POINT

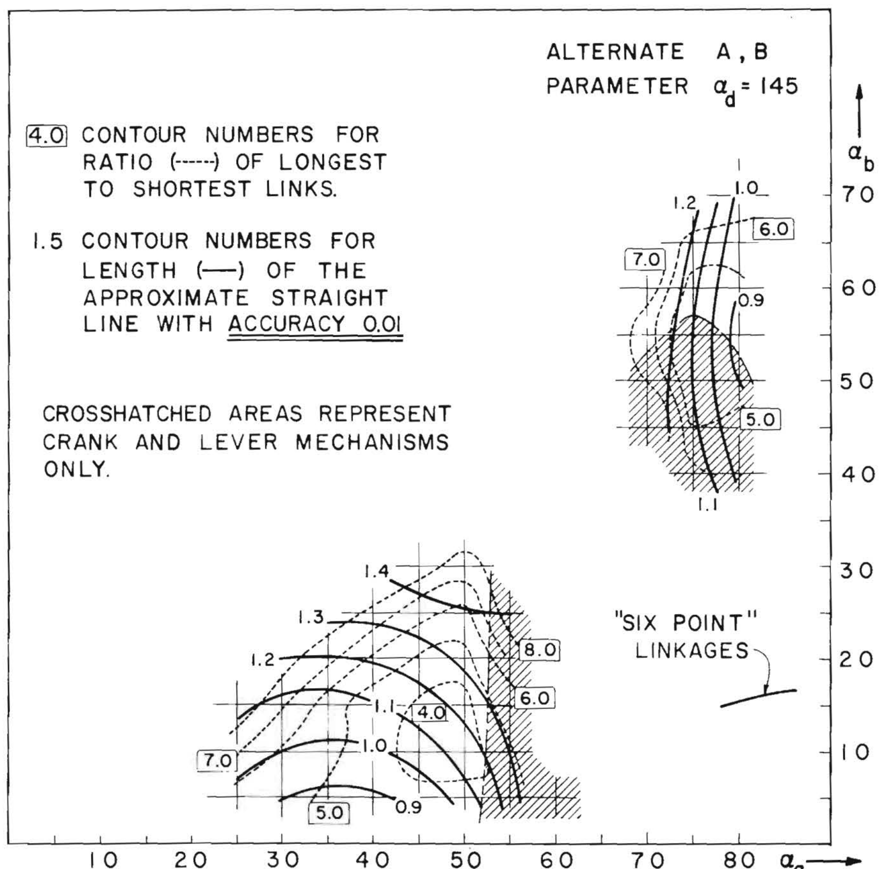


CHART BB 11 LENGTH-RATIO

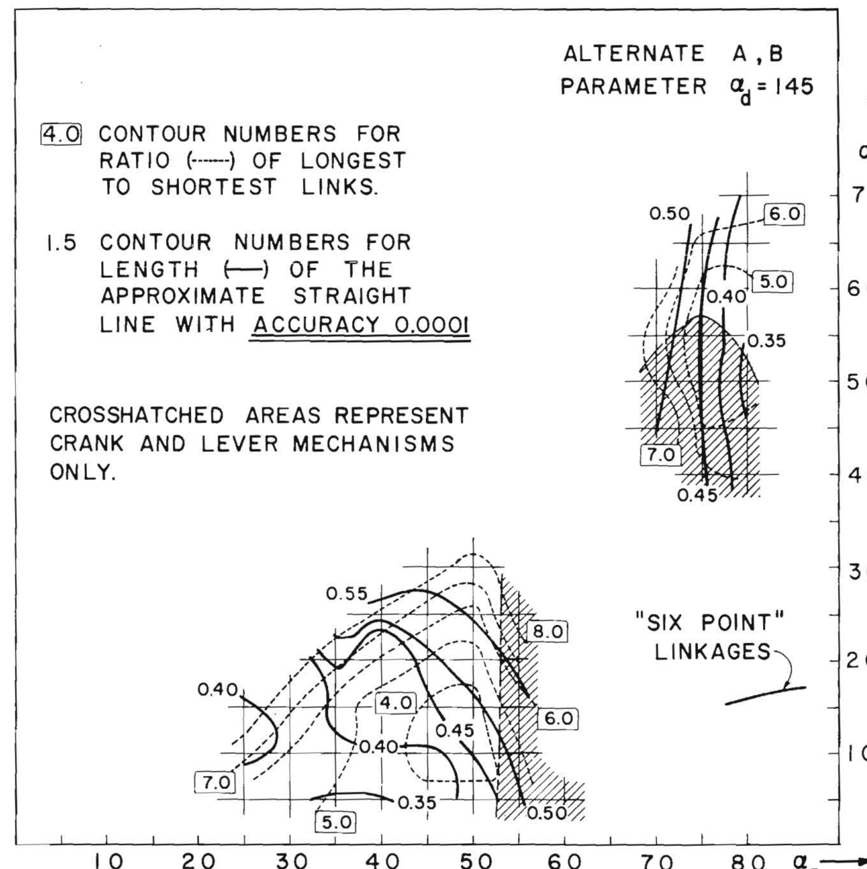


CHART BB 12 LENGTH-RATIO

BALL-BURMESTER POINT

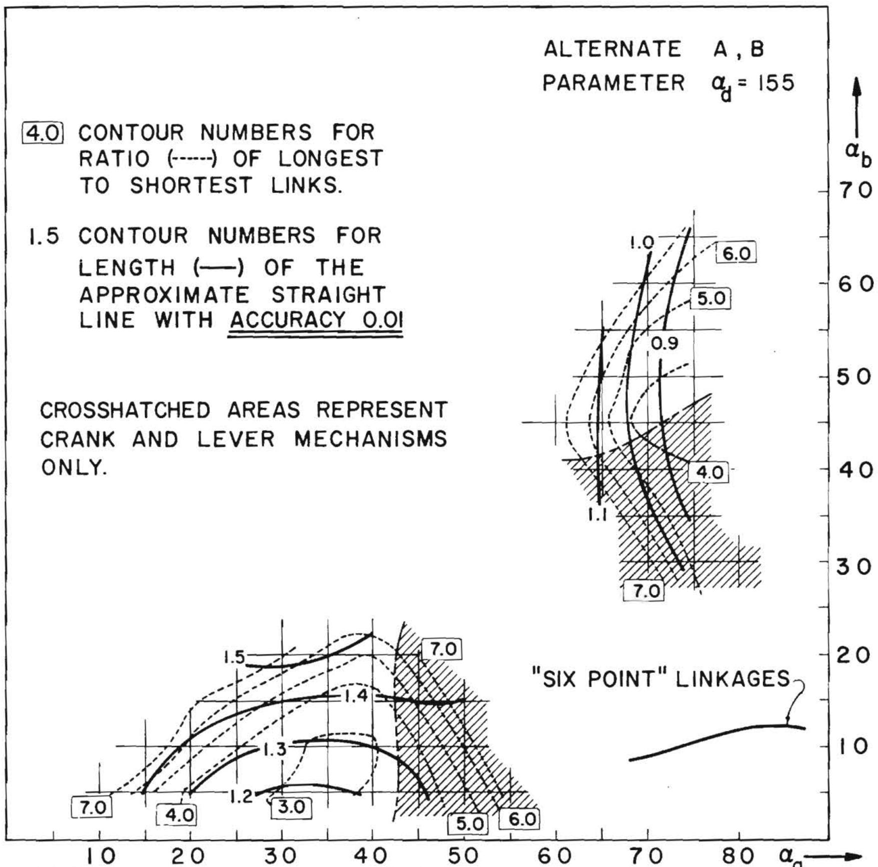


CHART BB 13 LENGTH-RATIO

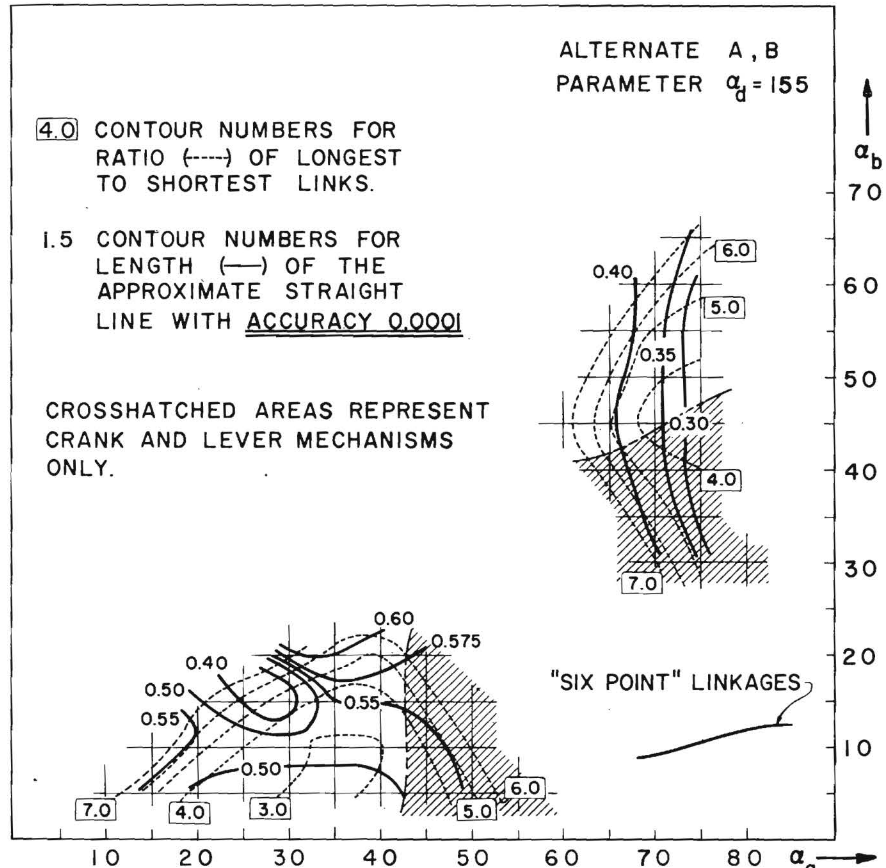


CHART BB 14 LENGTH-RATIO

BALL-BURMESTER POINT

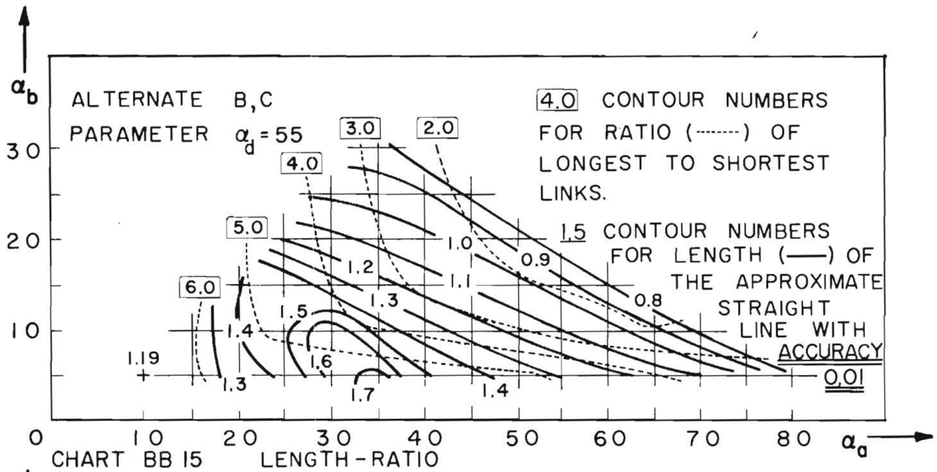


CHART BB 15 LENGTH-RATIO

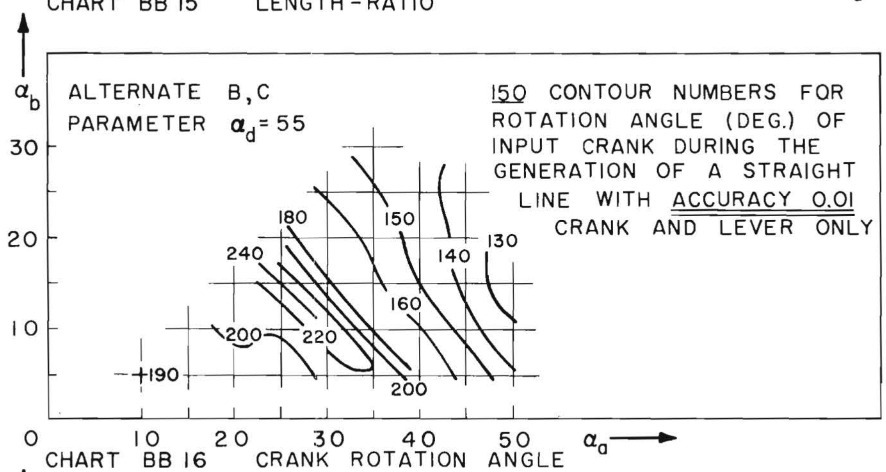


CHART BB 16 CRANK ROTATION ANGLE

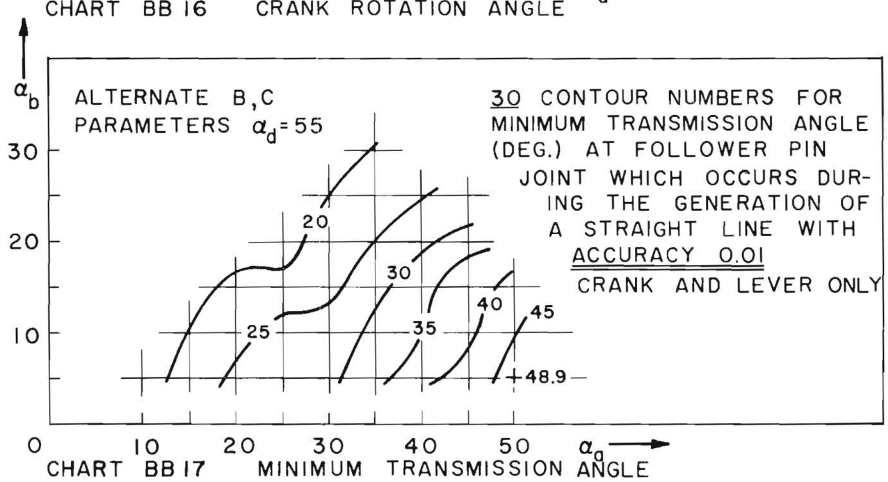


CHART BB 17 MINIMUM TRANSMISSION ANGLE

BALL–BURMESTER POINT

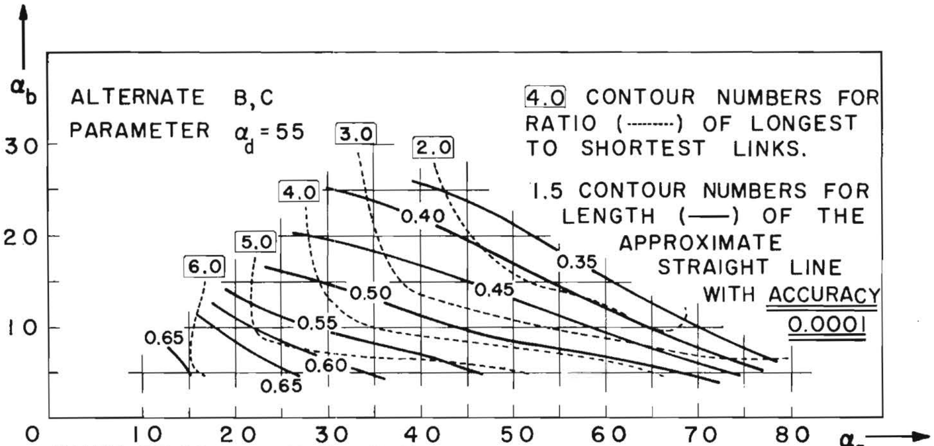


CHART BB 18 LENGTH-RATIO

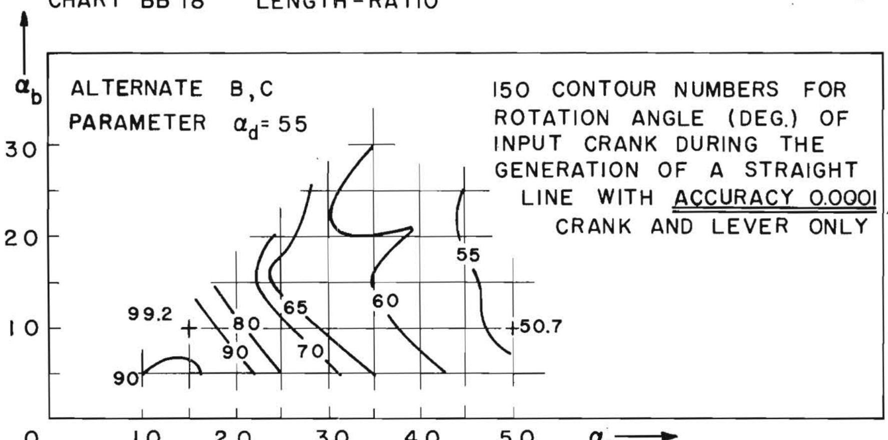


CHART BB 19 CRANK ROTATION ANGLE

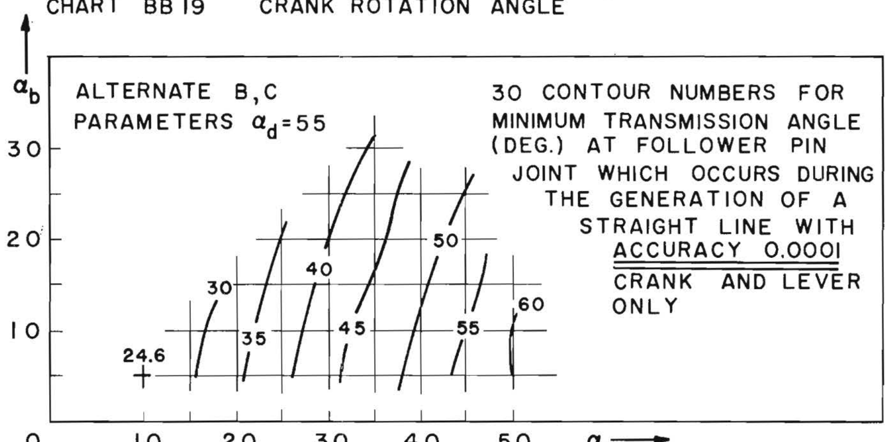


CHART BB 20 MINIMUM TRANSMISSION ANGLE

BALL-BURMESTER POINT

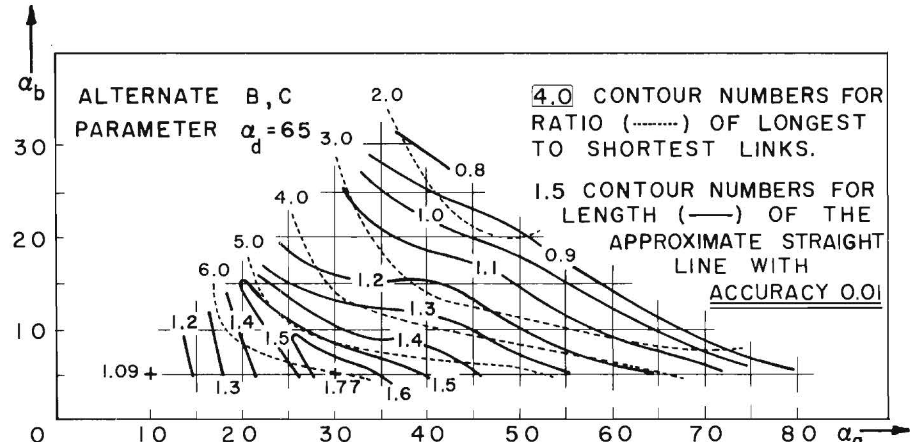


CHART BB 21 LENGTH-RATIO

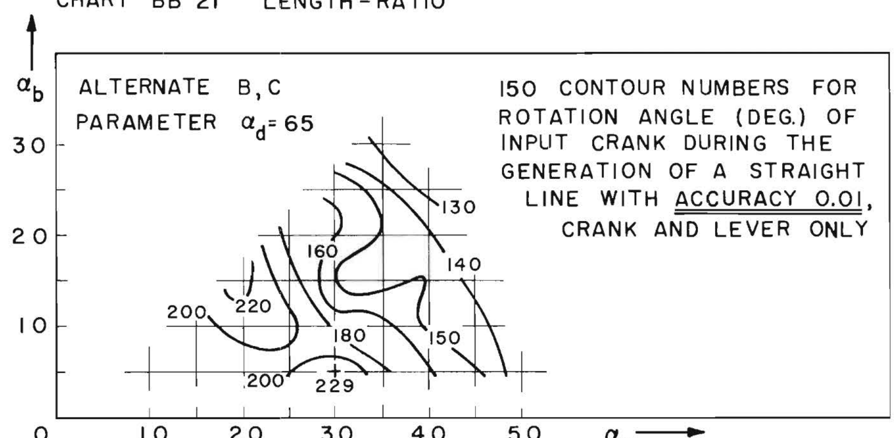


CHART BB 22 CRANK ROTATION ANGLE

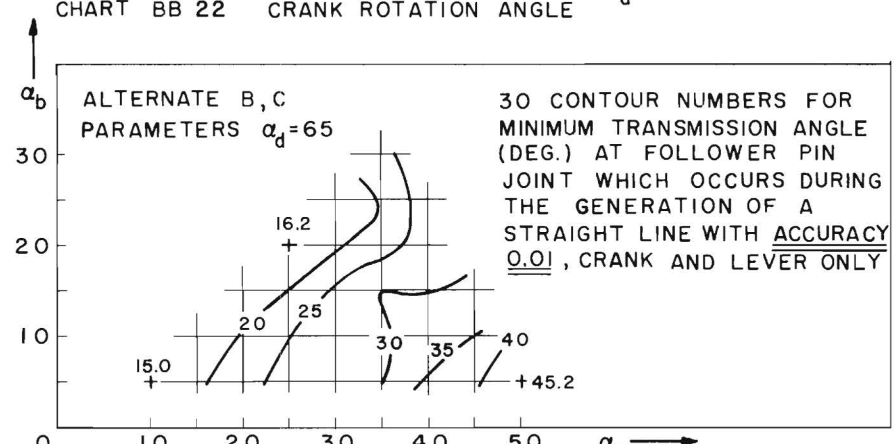


CHART BB 23 MINIMUM TRANSMISSION ANGLE

BALL-BURMESTER POINT

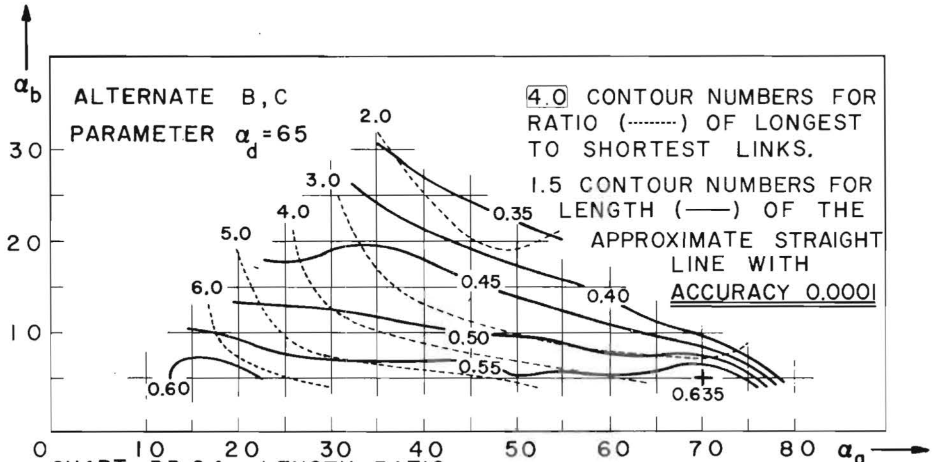


CHART BB 24 LENGTH-RATIO

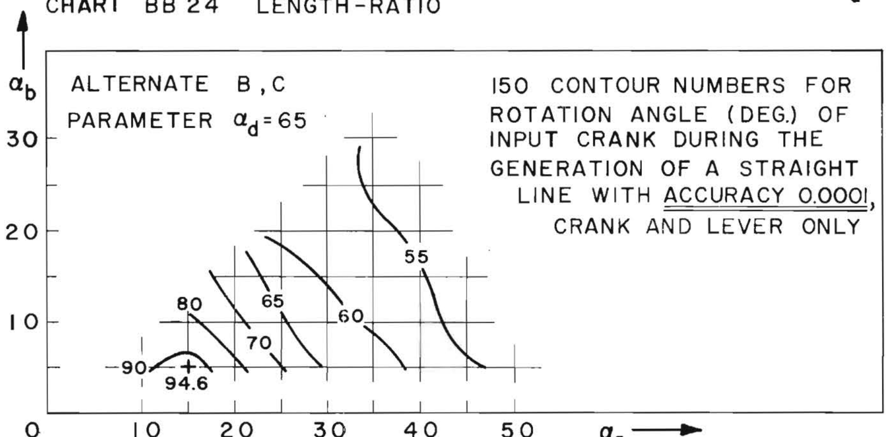


CHART BB 25 CRANK ROTATION ANGLE

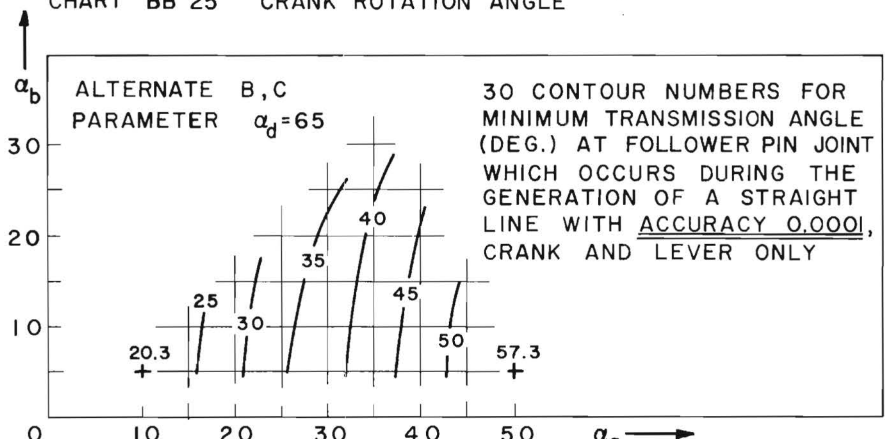


CHART BB 26 MINIMUM TRANSMISSION ANGLE

BALL-BURMESTER POINT

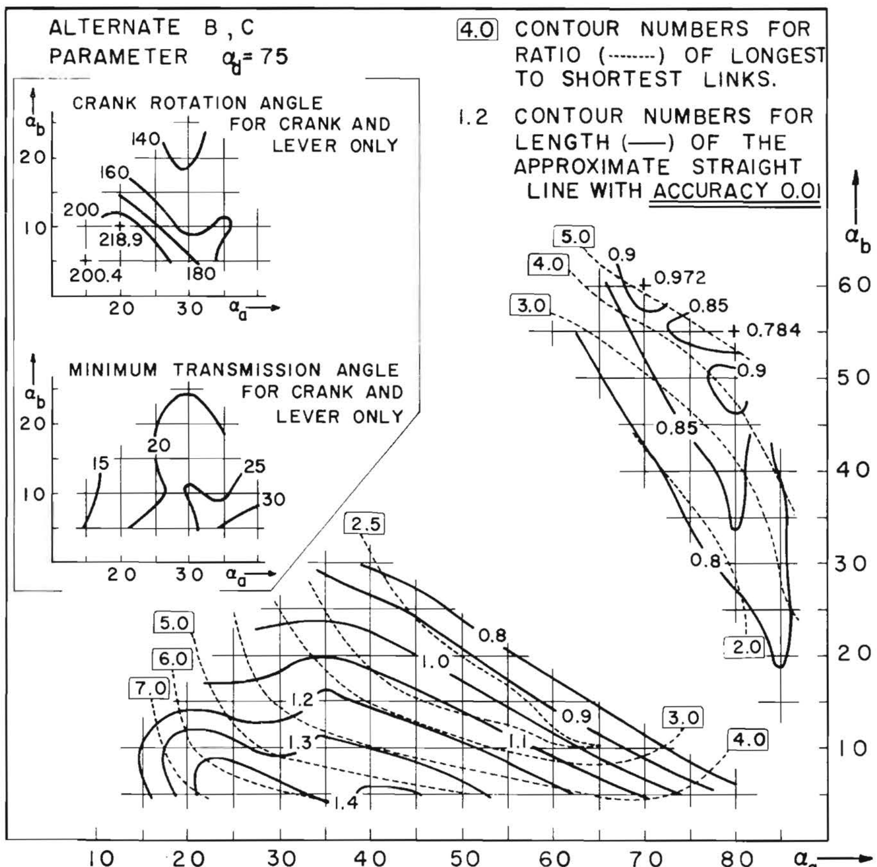


CHART BB 27 LENGTH-RATIO

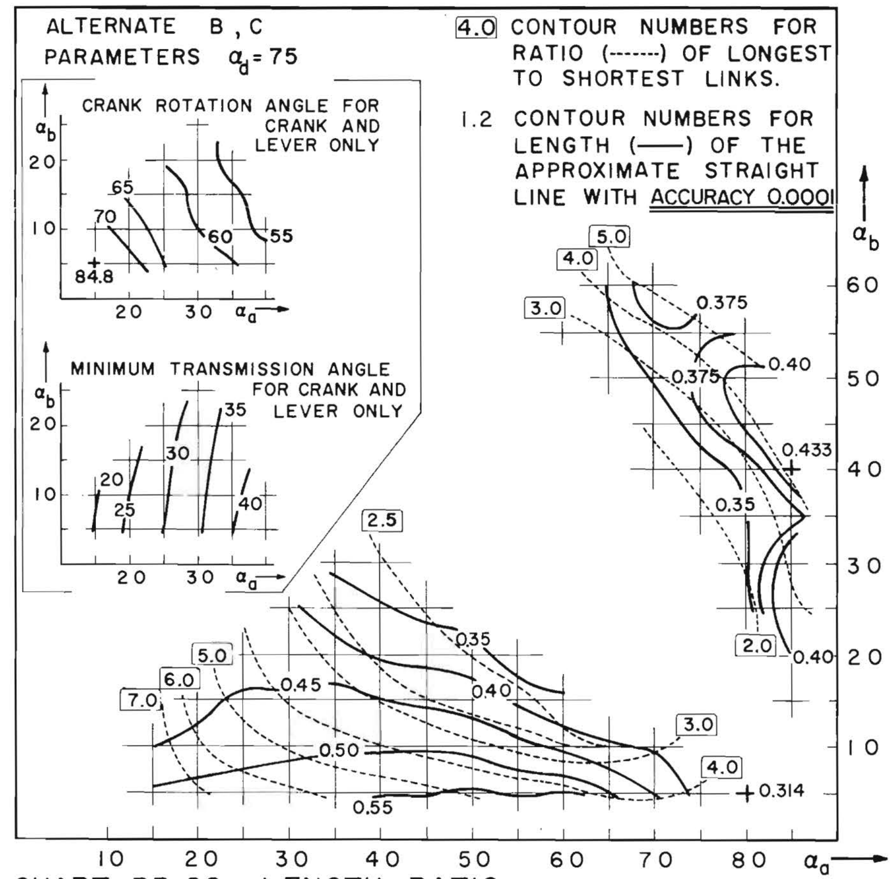


CHART BB 28 LENGTH-RATIO

BALL-BURMESTER POINT

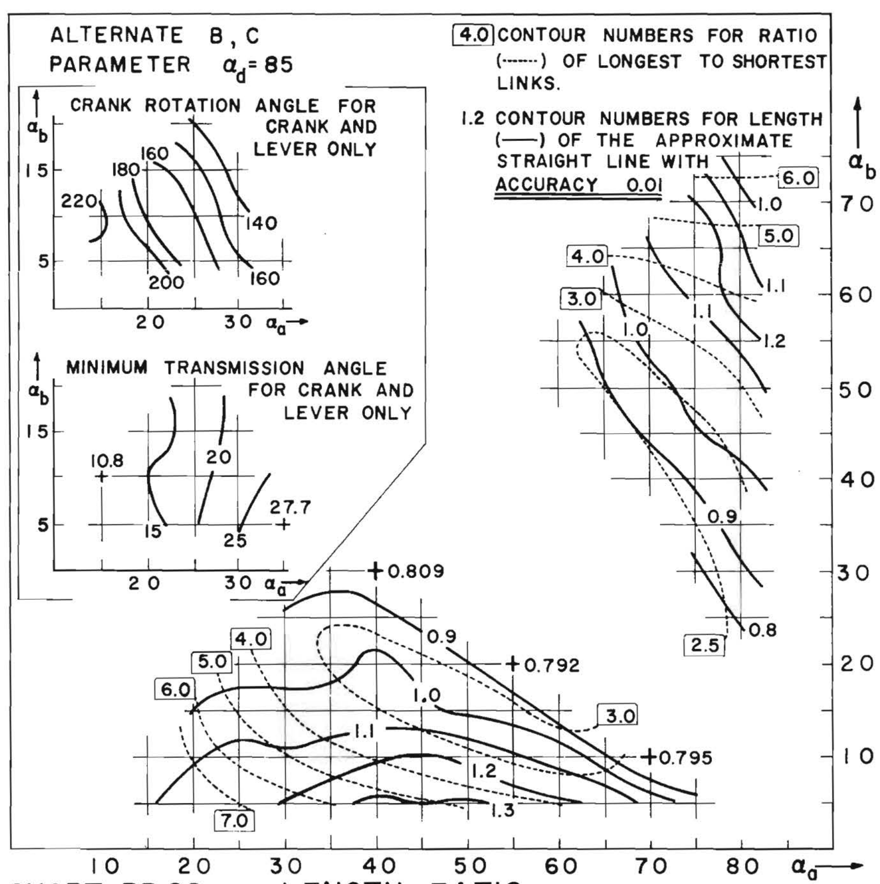


CHART BB 29 LENGTH-RATIO

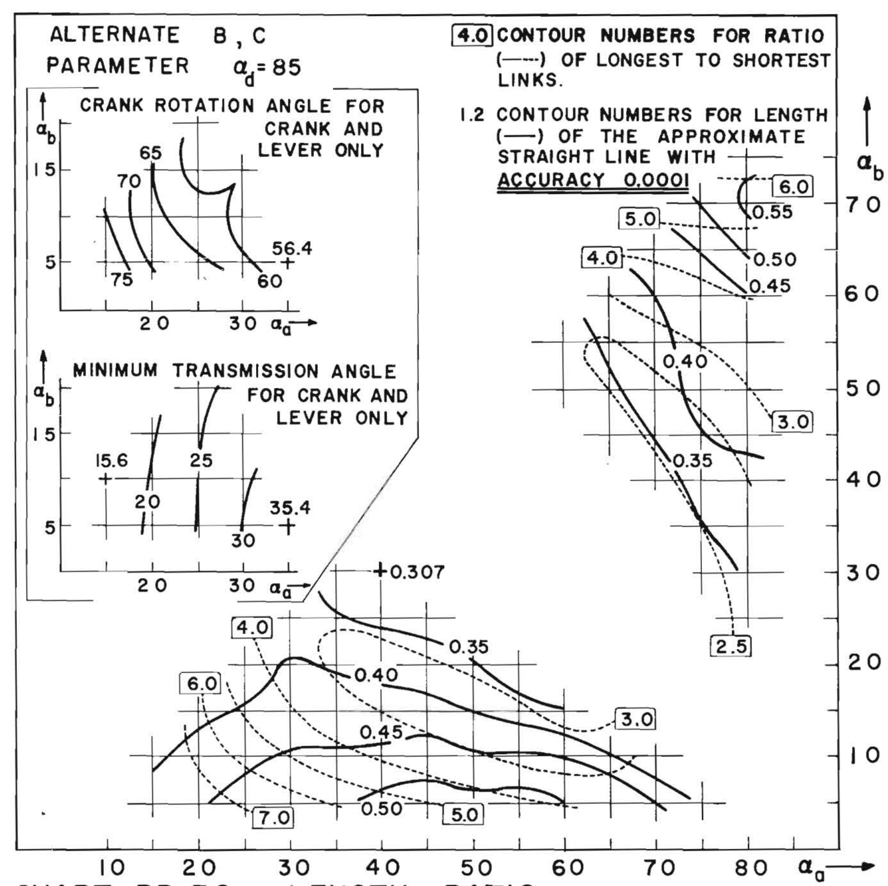


CHART BB 30 LENGTH-RATIO

BALL-BURMESTER POINT

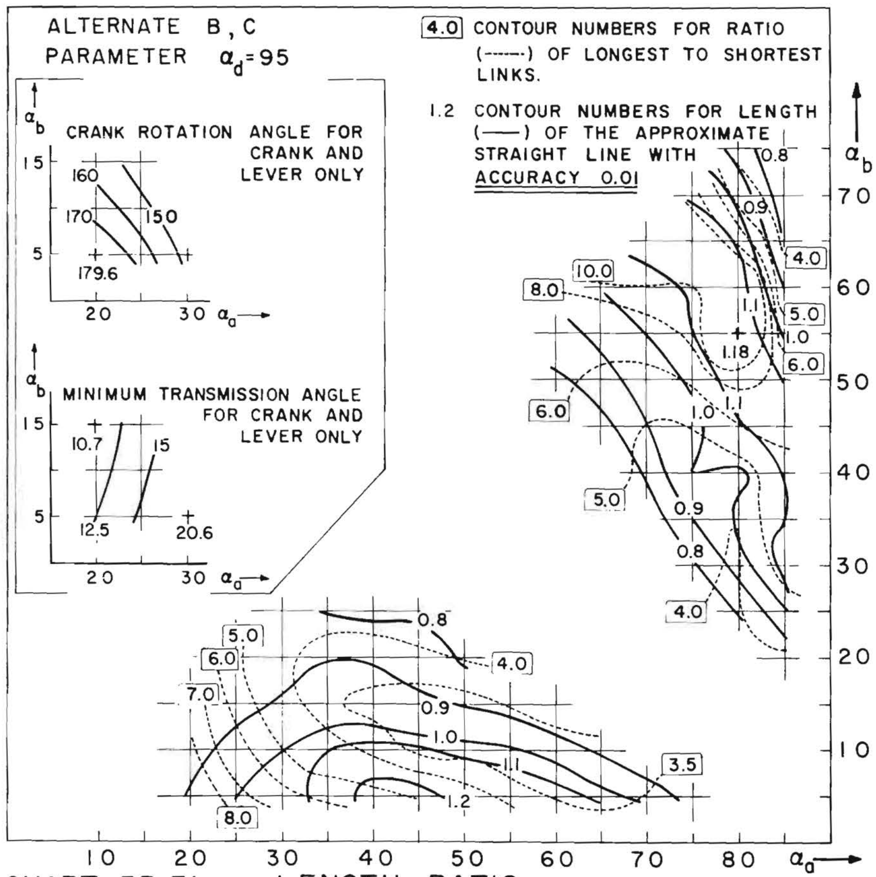


CHART BB 31 LENGTH-RATIO

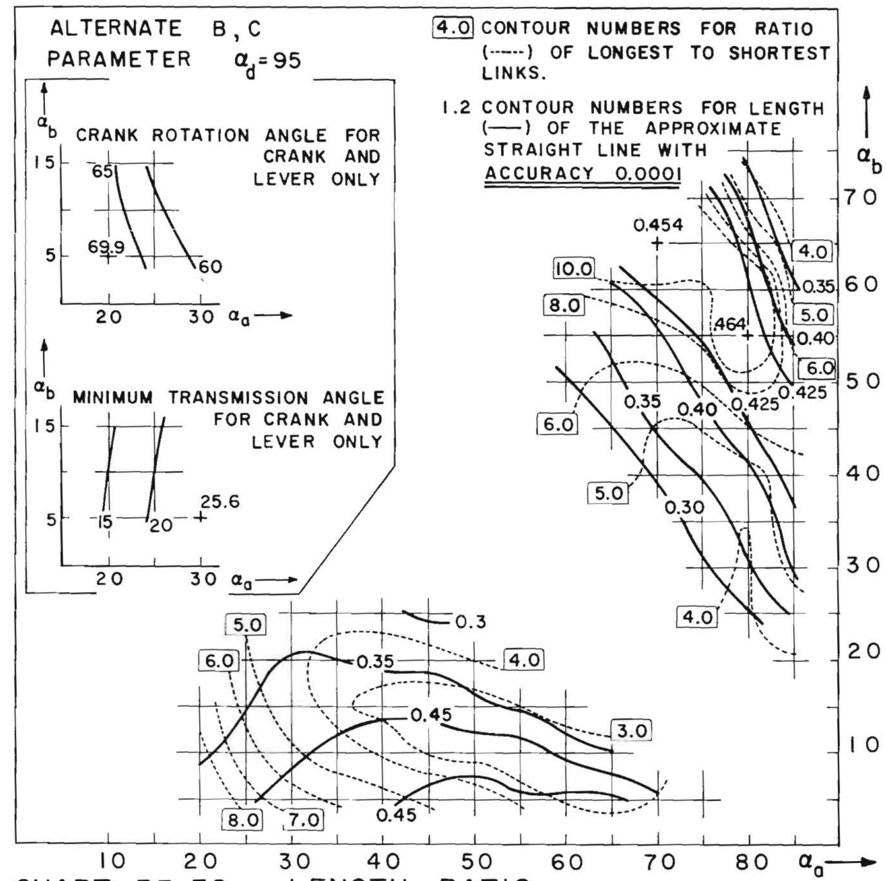


CHART BB 32 LENGTH-RATIO

BALL-BURMESTER POINT

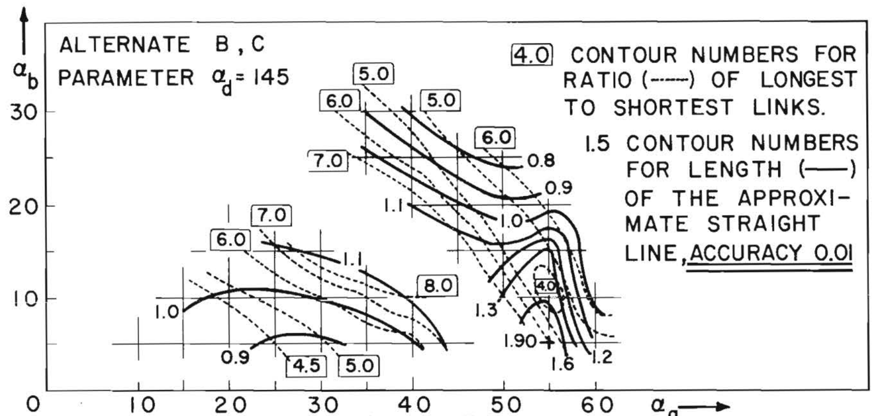


CHART BB 33

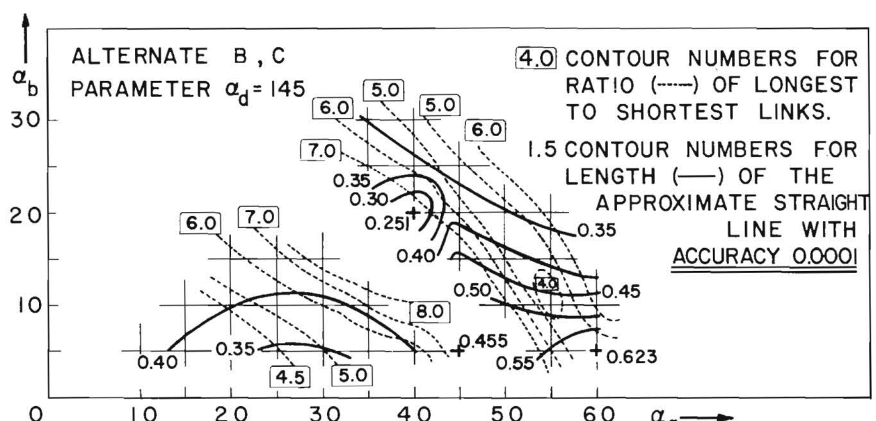


CHART BB 34

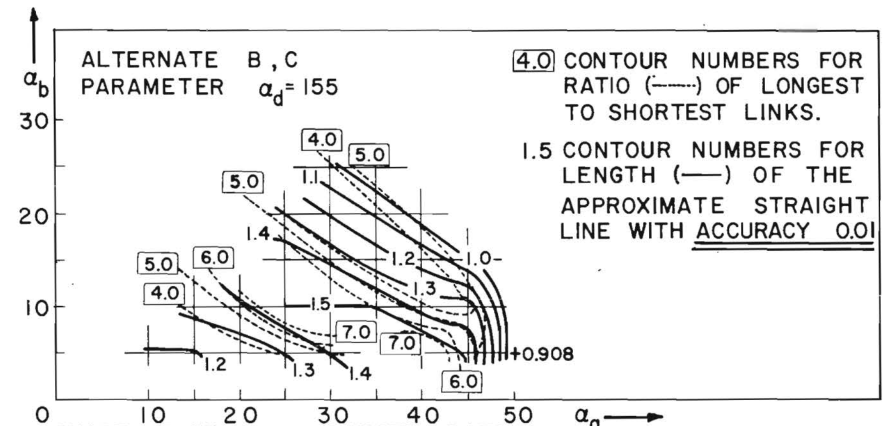


CHART BB 35

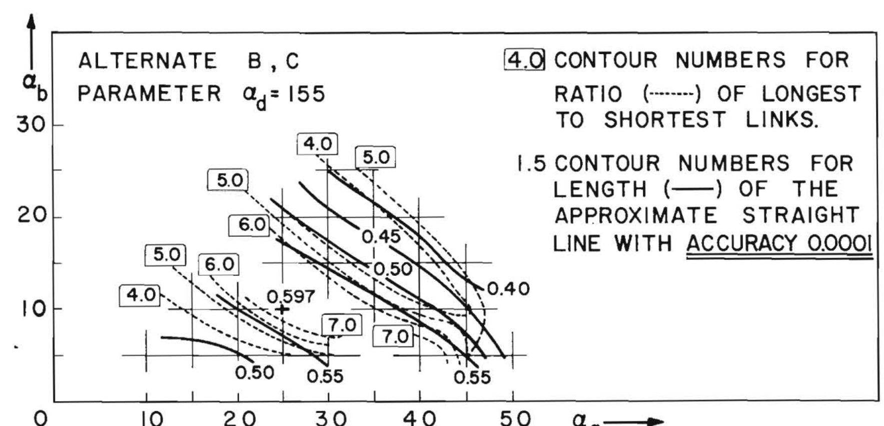


CHART BB 36

The Ball-Burmester at Inflection Pole Case

Charts IP1-IP3 represent linkages based on cranks O_aA and O_bB, IP4-IP6 contain the linkages having cranks O_bB and O_cC, and IP7-IP9 are of cranks O_cC and O_aA. Charts IP-1, 4 and 7 give the ratio of longest to shortest link lengths and IP-2, 5 and 8 present the straight-line outputs to the accuracy of 0.01. The remaining charts concern the 0.0001 accuracy.

The set of charts in this part derive from the situation where the Ball point not only coincides with a Burmester point, but, in addition, the combination occurs at the inflection pole (J in Fig. 6). Design variables are reduced to two and the equations simplified.

The coordinates of the output point $\underline{D}$ (Fig. 6) are $\alpha_d = 90^\circ$ and PD = 1.0 by definition of the case. Fixed pivot O_a and pin joint A are determined using

$$PO_a = k \sin \alpha_a \quad , \quad PA = \frac{PO_a}{k + 1} \tag{8}$$

where $\underline{k}$ and α_a are the design parameters (selected by designer).

The crank pole ray is located by

$$\alpha_b = \arctan \left\{ - \frac{k + 1}{k + 3} \; \frac{1}{\tan \alpha_a} \right\} \tag{9}$$

while fixed pivot O_b and pin joint $\underline{B}$ are at

$$PO_b = k \sin \alpha_b \quad , \quad PB = \frac{PO_b}{k + 1} \tag{10}$$

Eqs. (8) to (10) determine linkage O_aABO_b. A third crank CO_c may also be found and interchanged with AO_a or BO_b to form alternate linkages with similar

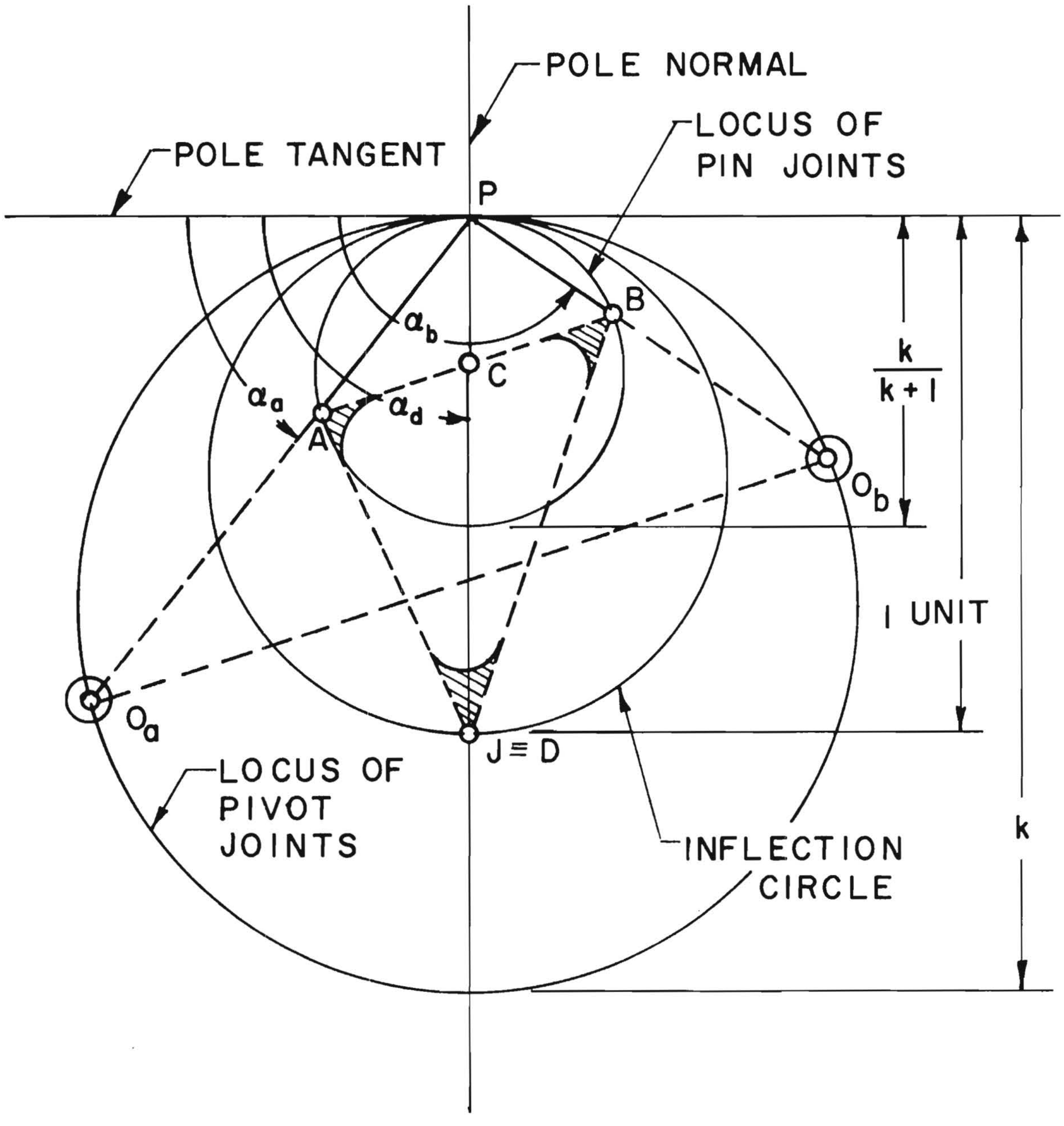


Figure 6. Ball Burmester at the Inflection Pole.

$\underline{D}$ outputs.

$$PC = \frac{k}{2k + 4} \quad , \quad PO_c = \frac{k}{k + 4} \tag{11}$$

At this point the linkage should be laid out graphically. If points A, B, and C are collinear and AB is parallel to O_aO_b, the computation is probably correct. The following list of equations need be considered only after a linkage has been chosen which suits the designers requirements.

The length of the fixed links are

$$O_aO_b = k \left\{ \sin^2\alpha_a + \sin^2\alpha_b - 2 \sin \alpha_a \sin \alpha_b \cos (\alpha_b - \alpha_a) \right\}^{\frac{1}{2}}$$

$$O_aO_c = \frac{k}{k + 4} \left\{ 1 + (k^2 + 6k + 8) \sin^2\alpha_a \right\}^{\frac{1}{2}} \tag{12}$$

$$O_bO_c = \frac{k}{k + 4} \left\{ 1 + (k^2 + 6k + 8) \sin^2\alpha_b \right\}^{\frac{1}{2}}$$

and the dimensions of the cranks are

$$O_aA = \frac{k^2}{k + 1} \sin \alpha_a$$

$$O_bB = \frac{k^2}{k + 1} \sin \alpha_b \tag{13}$$

$$O_cC = \frac{k^2/2}{(k + 2)(k + 4)}$$

The length of the various coupler dimensions are given by

$$AB = O_aO_b/(k + 1)$$

$$AD = M = \frac{1}{k + 1}\left\{ (k + 1)^2 - (k^2 + 2k)\sin^2\alpha_a \right\}^{\frac{1}{2}}$$

$$BD = N = \frac{1}{k + 1}\left\{ (k + 1)^2 - (k^2 + 2k)\sin^2\alpha_b \right\}^{\frac{1}{2}}$$

$$CD = \frac{k + 4}{2k + 4} \tag{14}$$

$$CB = \frac{k}{(k + 1)(k + 2)}\left\{\frac{1}{4}(k + 1)^2 + (k + 2)\sin^2\alpha_b \right\}^{\frac{1}{2}}$$

$$CA = AB - CB$$

Example

The method of calculation and utilization of the charts is illustrated by choosing, as an example, the design parameters

$$\alpha_a = 25^\circ \quad , \quad k = -2.2$$

For these values, the ratio of link lengths (Chart IP1) is approximately 3.5 and the approximate straight line output is 1.0 unit long for an accuracy of 0.01 unit (Chart IP2). Eq. (8) gives

$$PO_a = -(2.2)(0.4226) = -0.930 \text{ unit}$$

$$PA = \frac{-0.930}{-1.2} = 0.775$$

and Eq. (9) yields

$$\alpha_b = \tan^{-1}\left\{ - \frac{-1.2}{(0.8)(0.4663)} \right\}$$

$$= \tan^{-1}(3.216) = 72.7^\circ$$

which in turn gives by Eq. (10)

$$PO_b = (-2.2)(0.9548) = -2.10$$

$$PB = \frac{-2.10}{-12} = 1.75$$

The third crank which lies along the pole normal is computed by means of Eq. (11) as

$$PC = \frac{-2.2}{-0.4} = 5.50$$

$$PO_c = \frac{-2.2}{1.8} = 1.22$$

When the above mechanism is laid out as below (Fig. 7), the points A, B, and C are collinear as required by theory and line AB is parallel to line O_aO_b. Therefore, the computations appear satisfactory.

Thus, proceed to calculate link dimensions, say, for the A,B linkage.

By equation (12), the fixed link becomes

$$O_aO_b = -2.2\left[(0.4226)^2 + (0.9548)^2 - 2(0.4226)(0.9548)(0.6730)\right]^{\frac{1}{2}}$$

$$= -2.2\,[0.735] = -1.61$$

Using Eqs. (13), the cranks are

$$O_aA = \frac{(-2.2)^2}{-1.2}\,(0.4226) = -1.70$$

$$O_bB = \frac{(-2.2)^2}{-1.2}\,(0.9548) = -3.85$$

and Eqs. (14), yield the coupler dimensions as

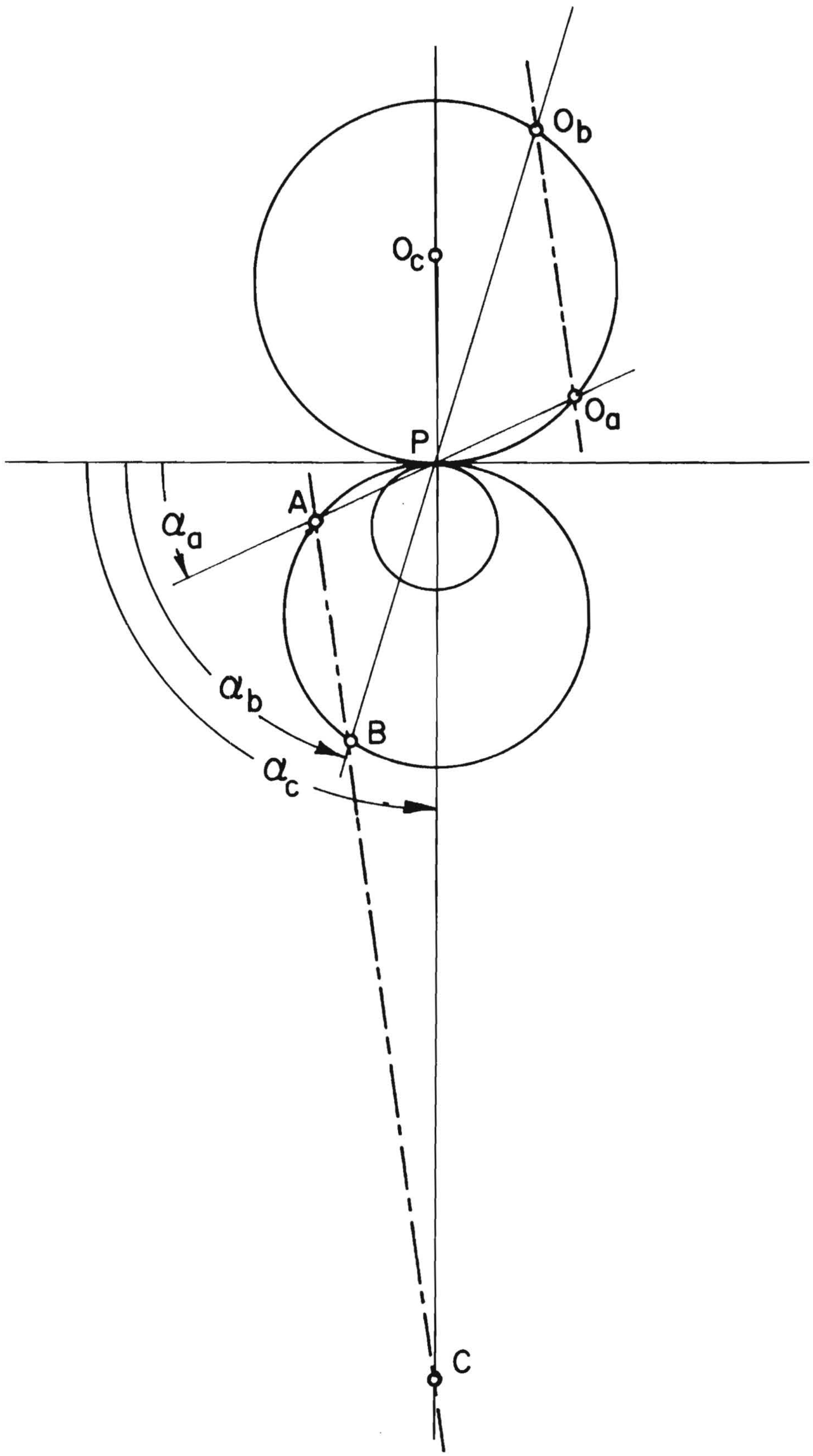


Figure 7. Collinearity of Burmester Points.

$$AB = S = (-1.61)/-1.2 = 1.34$$

$$AD = M = \frac{1}{-1.2}\left\{[-1.2]^2 - [(-2.2)^2 + 2(-2.2)][0.4226]^2\right\}^{\frac{1}{2}}$$

$$= (-0.834)(1.16) = -0.965$$

$$BD = N = \frac{1}{-1.2}\left\{[-1.2]^2 - [(-2.2)^2 + 2(-2.2)][0.9548]^2\right\}^{\frac{1}{2}}$$

$$= (-0.834)(0.996) = -0.830$$

The linkage as well as the path of point D are shown in Fig. 8 (note linkage is in mirror-image position to that of Fig. 7 and twice size - for convenience only). The unit length according to the above calculated dimensions is

$$UN = \frac{Q + R + S + T + \frac{M + N}{2}}{5}$$

$$= \frac{1.61 + 1.70 + 1.34 + 3.85 + \frac{0.965 + 0.830}{2}}{5}$$

$$= 1.88$$

In order to obtain correct application of the charts, it is important to note that the above size index is not unity (as required for comparative purposes on the charts). To get correct values for the length and error of the approximate straight-line output relative to the calculated size of the linkage, use

$$L_a = (L_c)(UN) = (1.0)(1.88) = 1.88$$

$$D_a = (D_c)(UN) = (0.01)(1.88) = 0.019$$

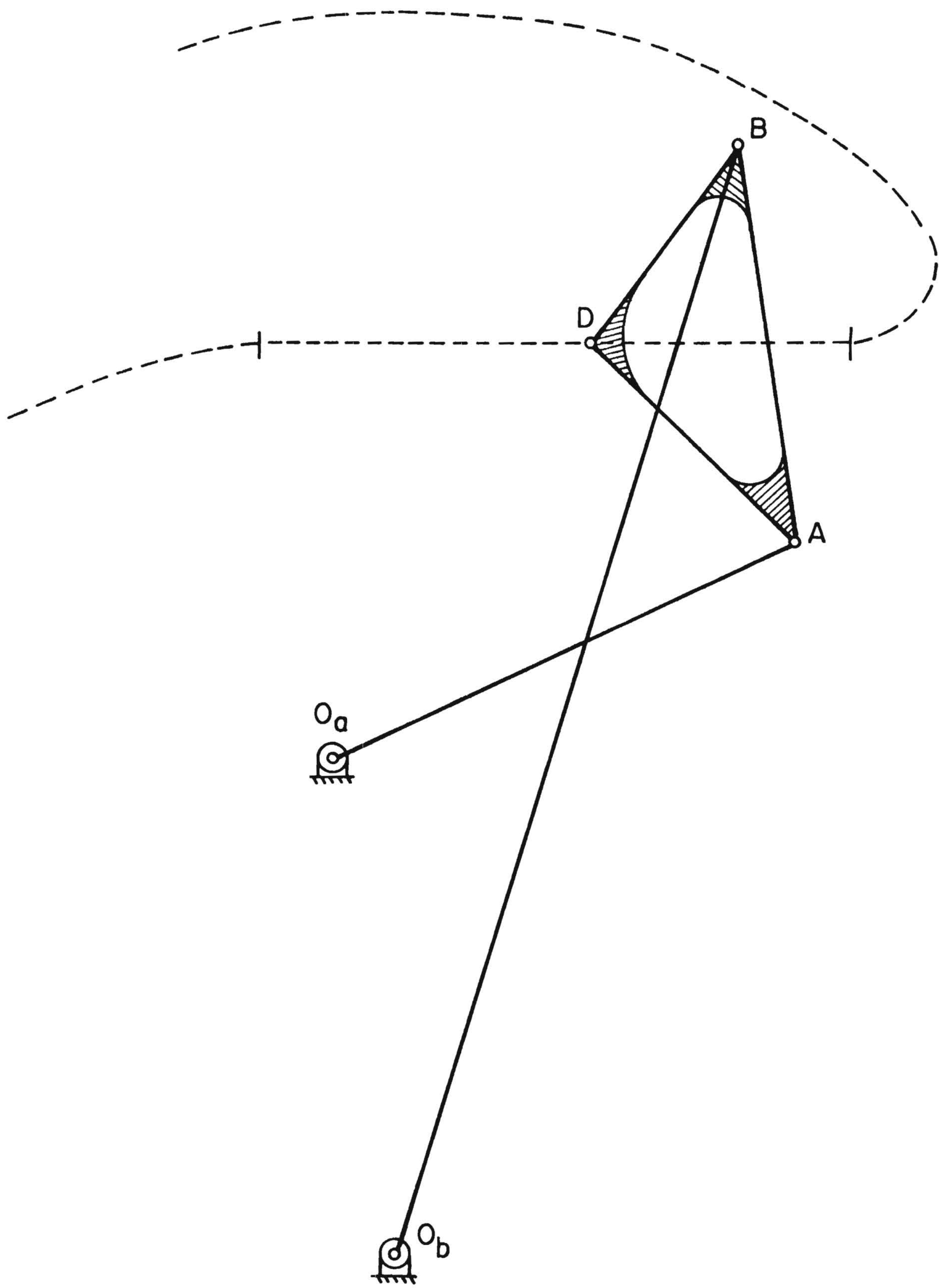


Figure 8. Example, Ball-Burmester at Inflection Pole.

where L_c and D_c are the chart values (Chart IP2).

Should the linkage be increased by a factor of 10, the dimensions (denoted by primes) are increased by a factor of 10. Note, then, that

$$L_a' = 18.8$$

$$D_a' = 0.19$$

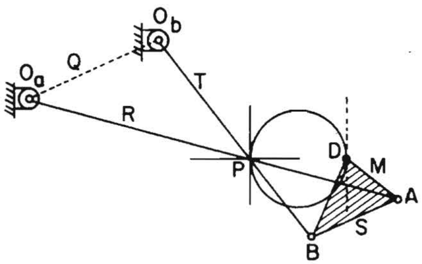
However, by considering Chart IP3, the values

$$L_a' = (10)(L_c)(UN) \approx (10)(0.4)(1.88) = 7.52$$

$$D_a' = (10)(D_c)(UN) = (10)(0.0001)(1.88) = 0.0019$$

are obtained for a shorter range of the coupler curve. These values indicate a very satisfactory accuracy and a sufficiently long output for many design purposes.

The other charts can be applied similarly for either of the alternate mechanisms O_aACO_c or O_bBCO_c.



BALL-BURMESTER POINT AT INFLECTION POLE

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME VALUE FOR THE RATIO OF LONGEST TO SHORTEST LINK COMPUTED THUS:

$$\text{RATIO} = \frac{\text{MAX.}\ (Q, R, S, T, \frac{M+N}{2})}{\text{MIN.}\ (Q, R, S, T, (M+N))}$$

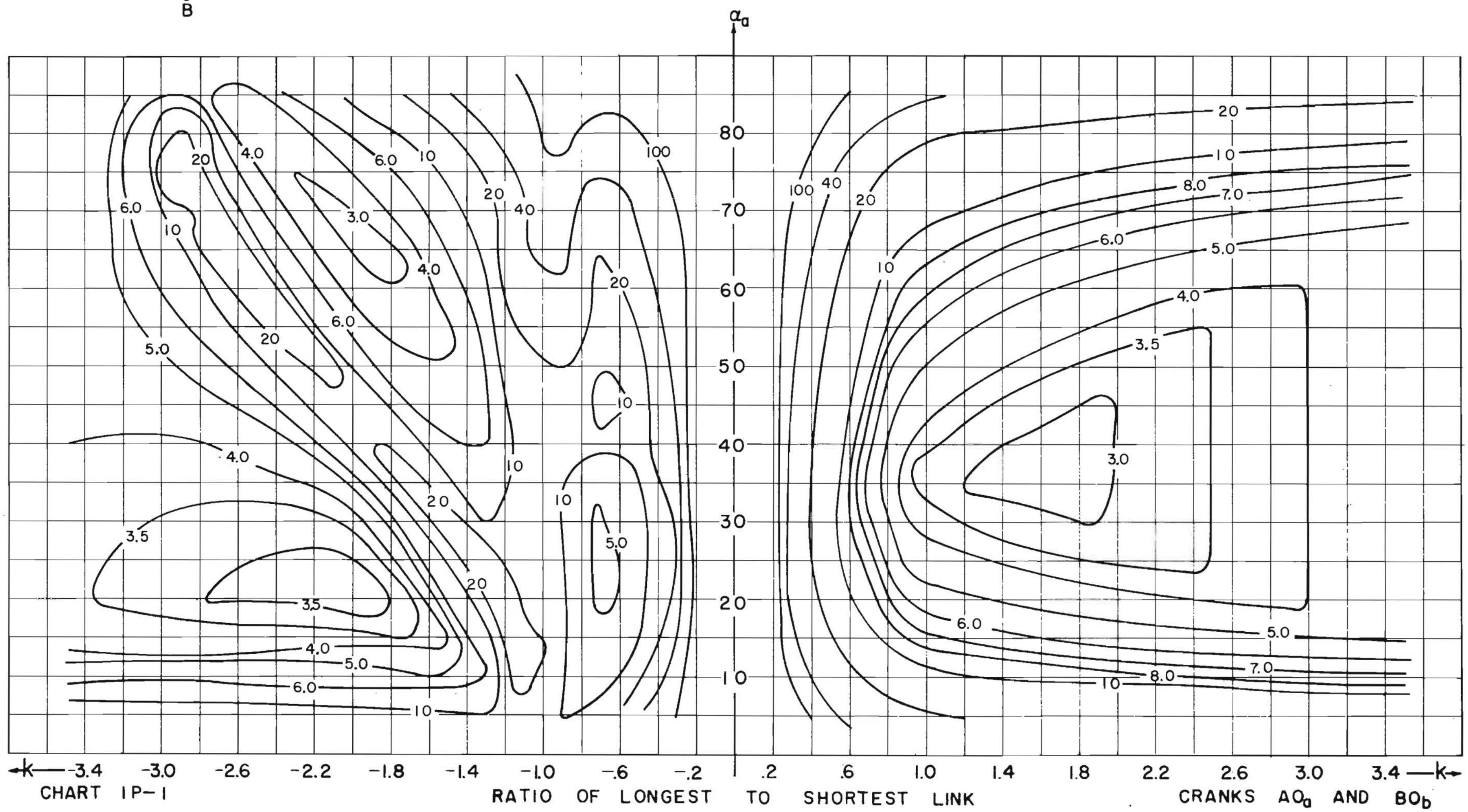
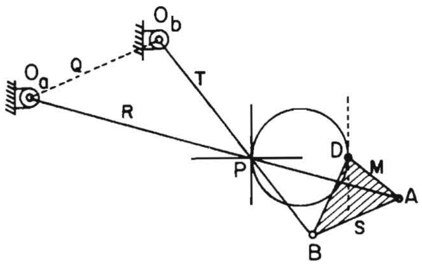


CHART IP-1



BALL–BURMESTER POINT AT INFLECTION POLE

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

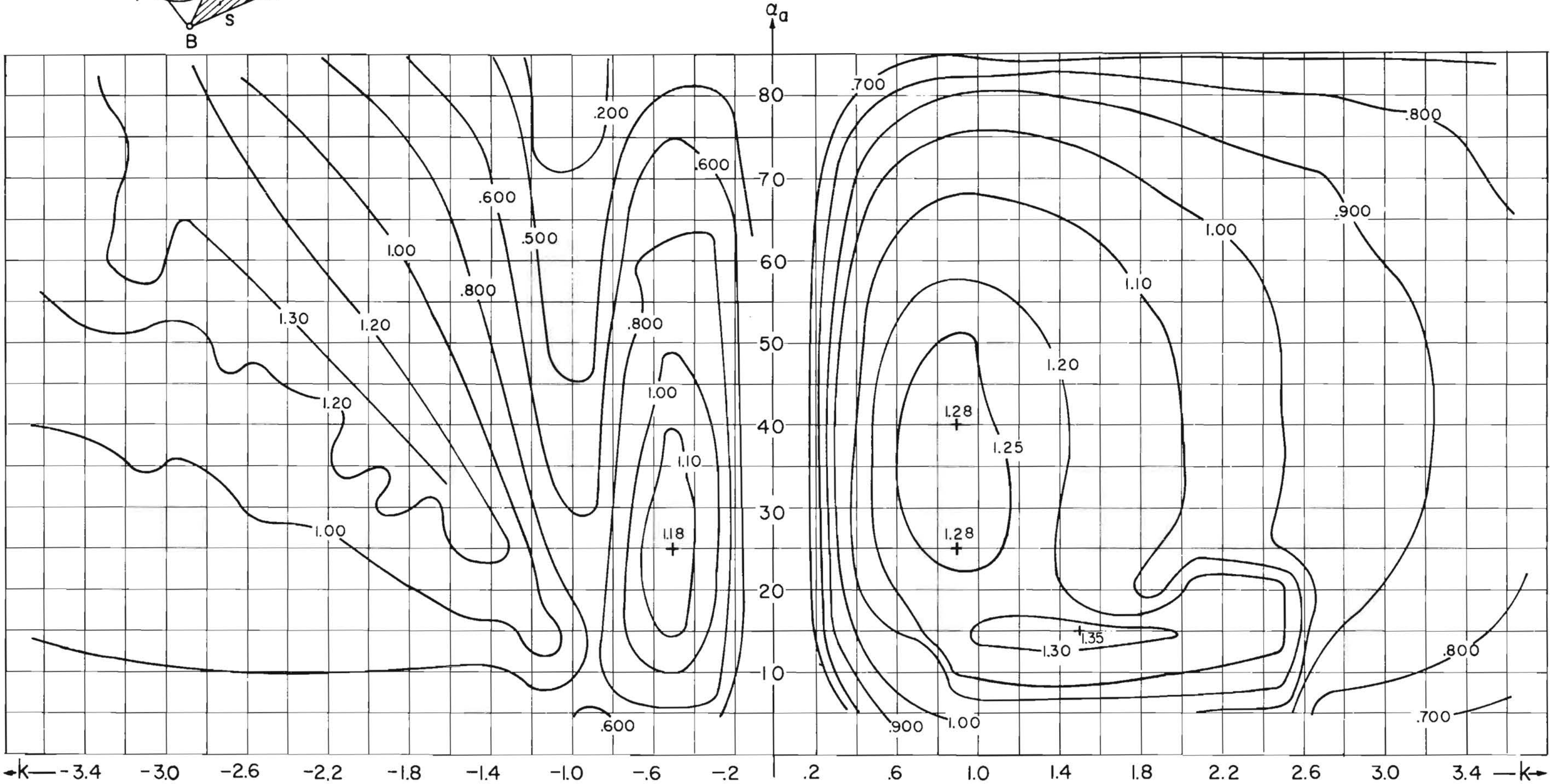
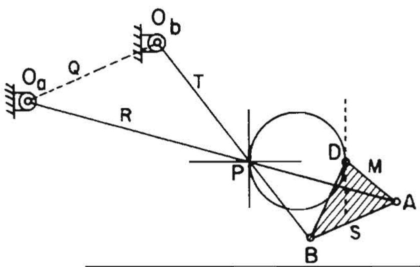


CHART IP–2

STRAIGHT LINE LENGTH L_c, DEVIATION $D_c = 0.01$ UNIT

CRANKS AO_a AND BO_b



BALL-BURMESTER POINT AT INFLECTION POLE

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

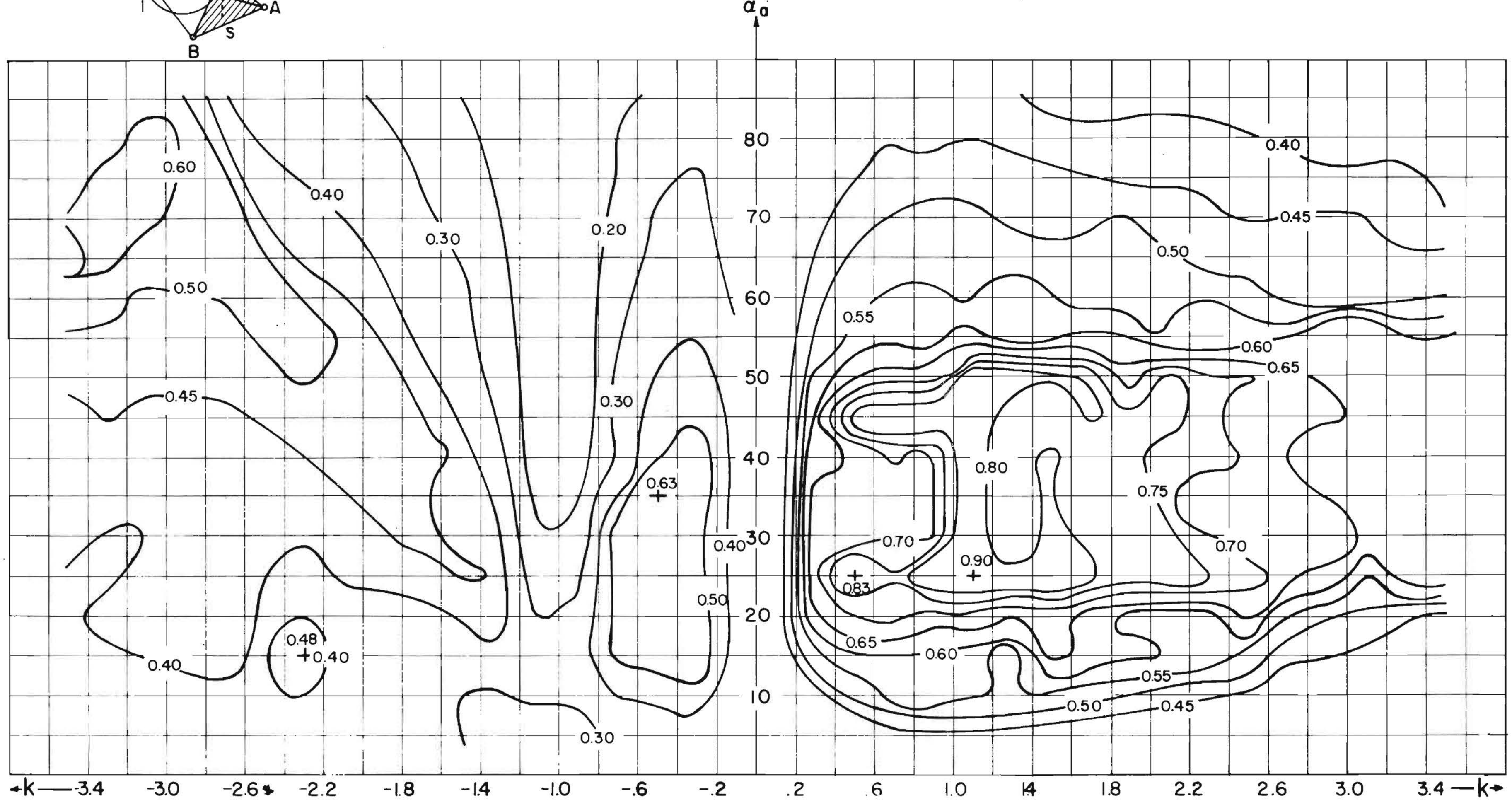
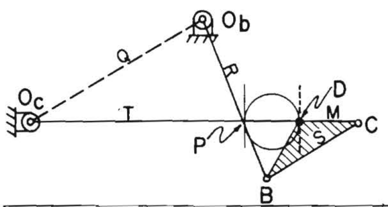


CHART IP-3

STRAIGHT LINE LENGTH L_c, DEVIATION $D_c = 0.0001$ UNIT

CRANKS AO_a AND BO_b



BALL-BURMESTER POINT AT INFLECTION POLE

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME VALUE FOR THE RATIO OF LONGEST TO SHORTEST LINK COMPUTED THUS:

$$\text{RATIO} = \frac{\text{MAX.}\ (Q, R, S, T, \frac{M+N}{2})}{\text{MIN.}\ (Q, R, S, T, (M+N))}$$

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISM

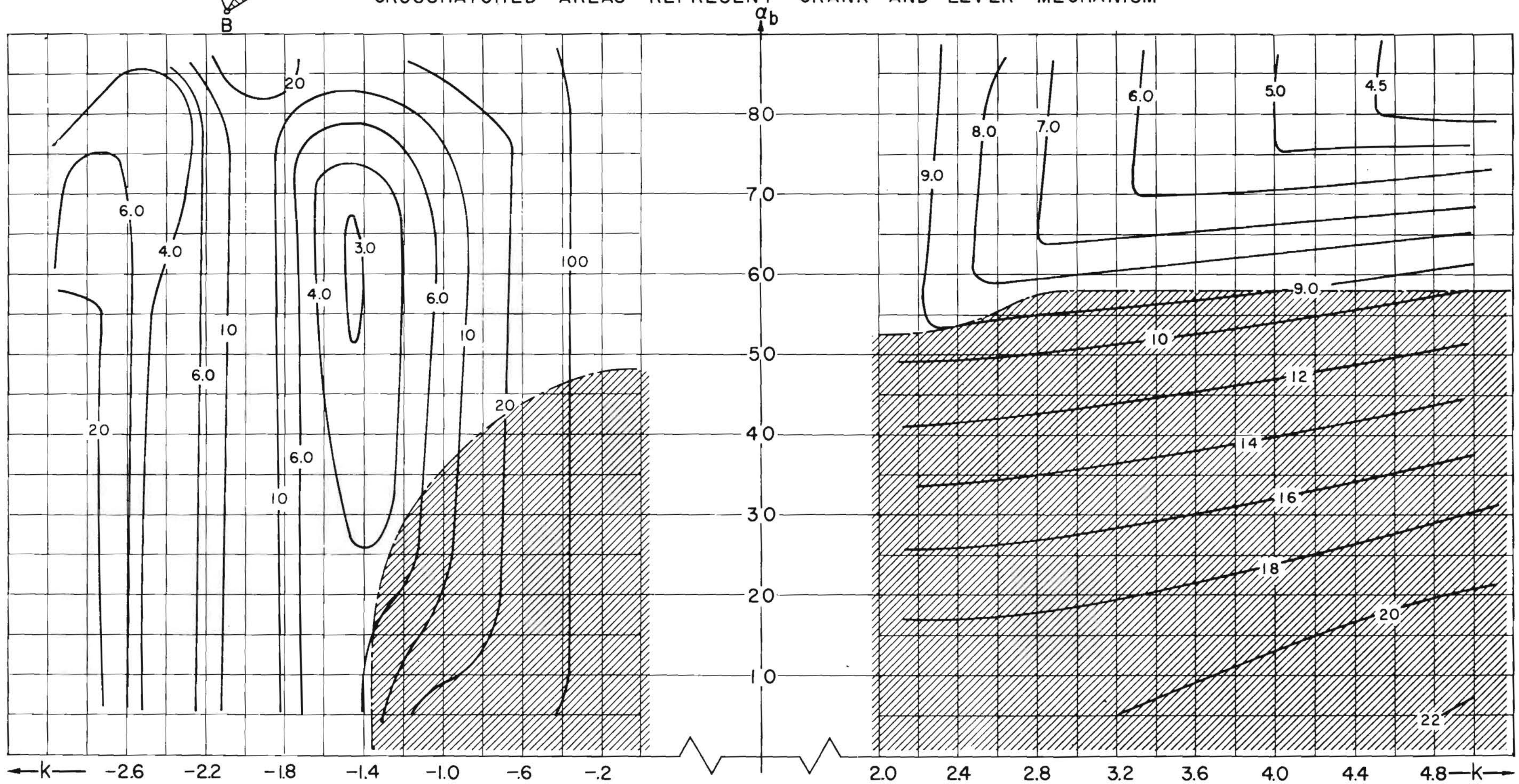
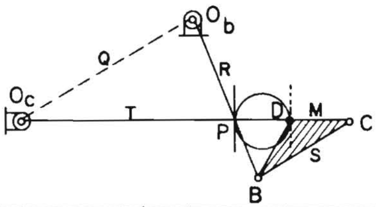


CHART IP-4

RATIO OF LONGEST TO SHORTEST LINK

CRANKS BO_b AND CO_c



BALL–BURMESTER POINT AT INFLECTION POLE

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISM

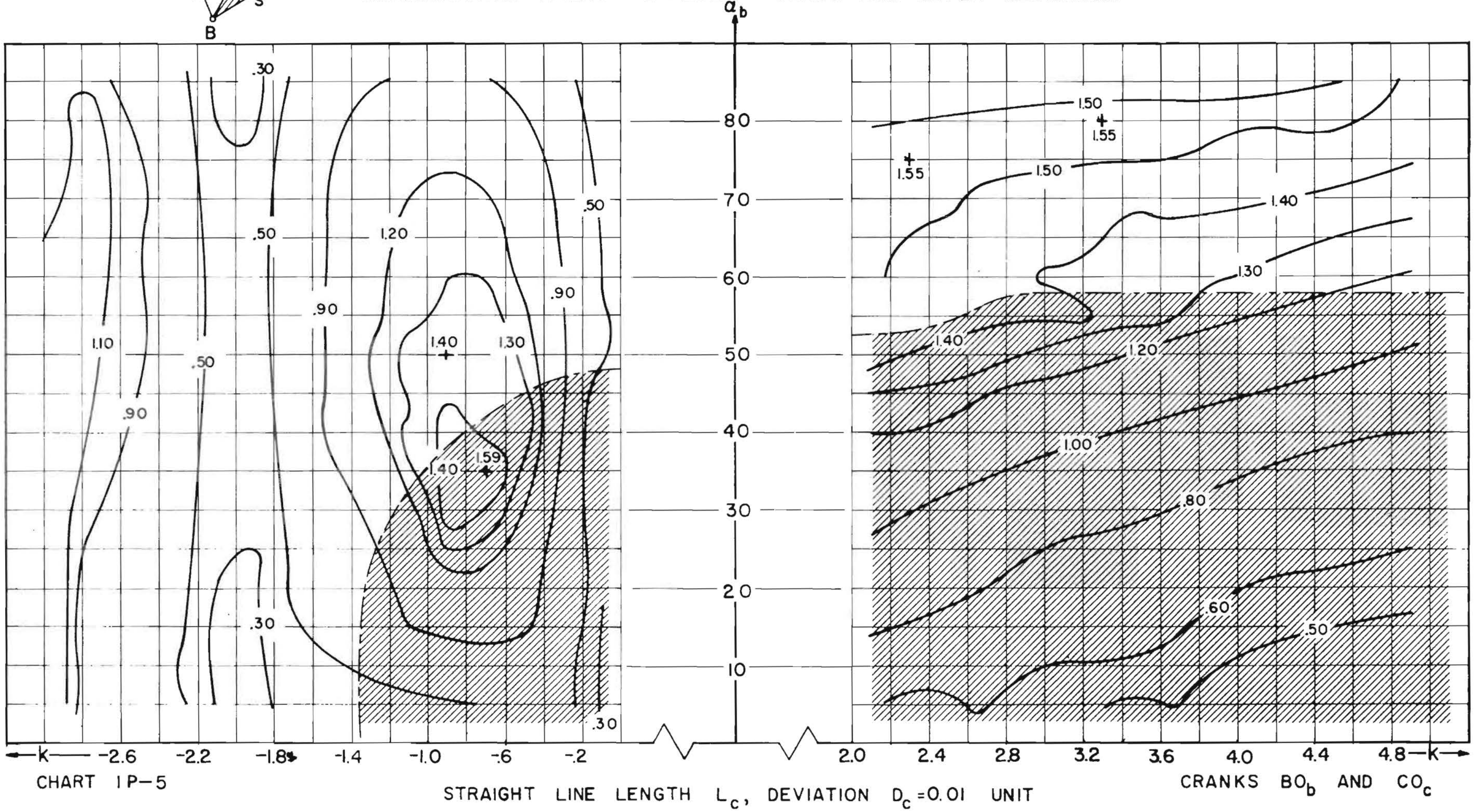
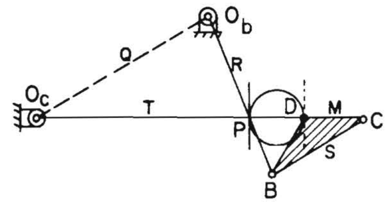


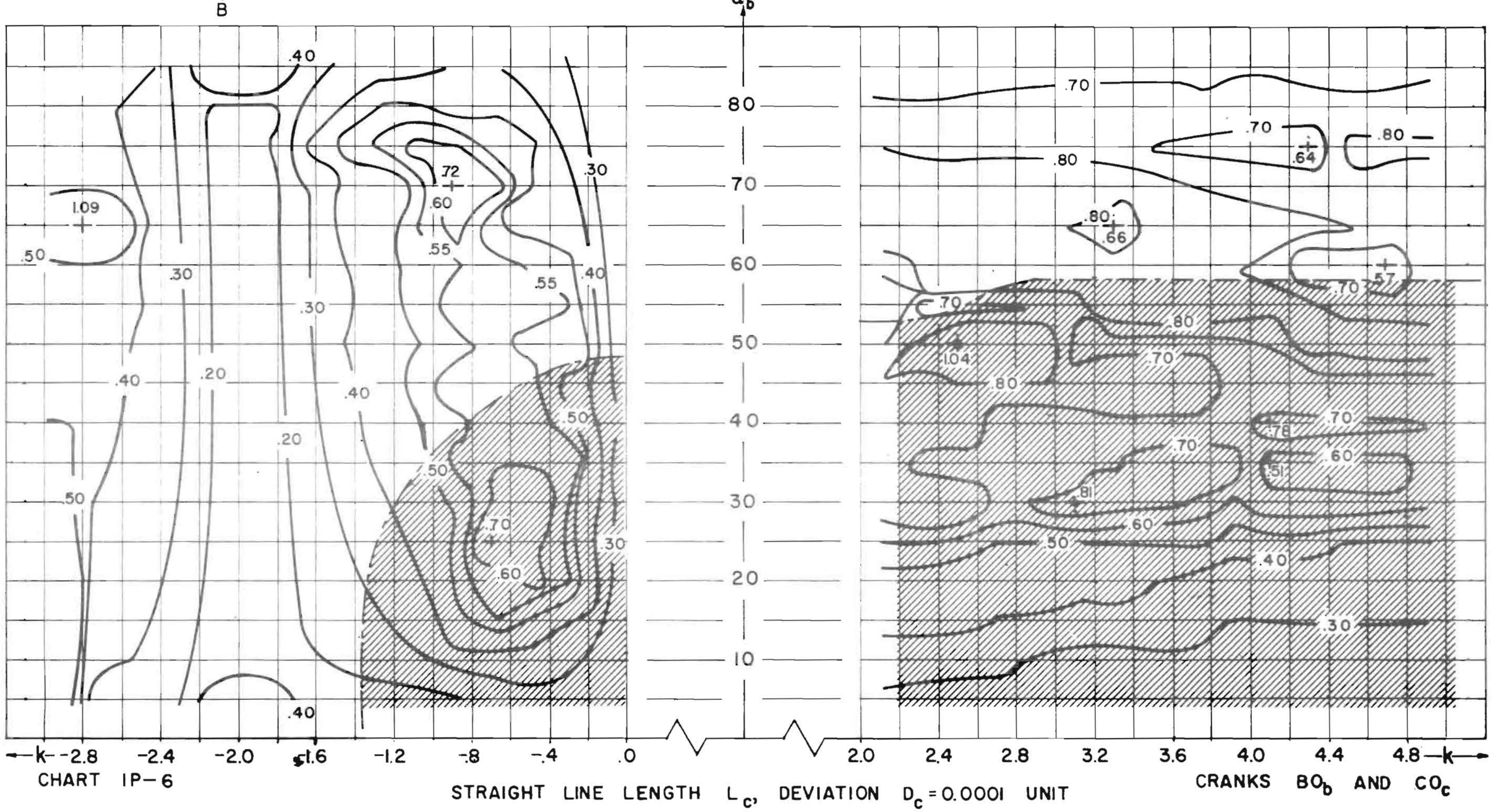
CHART IP–5

BALL-BURMESTER POINT AT INFLECTION POLE

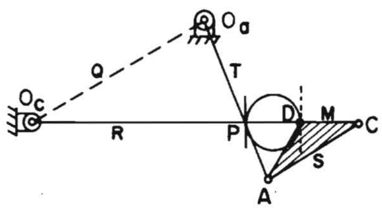


CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISM



BALL-BURMESTER POINT AT INFLECTION POLE



CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME VALUE FOR THE RATIO OF LONGEST TO SHORTEST LINK COMPUTED THUS:

$$\text{RATIO} = \frac{\text{MAX.}\left(Q, R, S, T, \frac{M+N}{2}\right)}{\text{MIN.}\left(Q, R, S, T, (M+N)\right)}$$

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISM

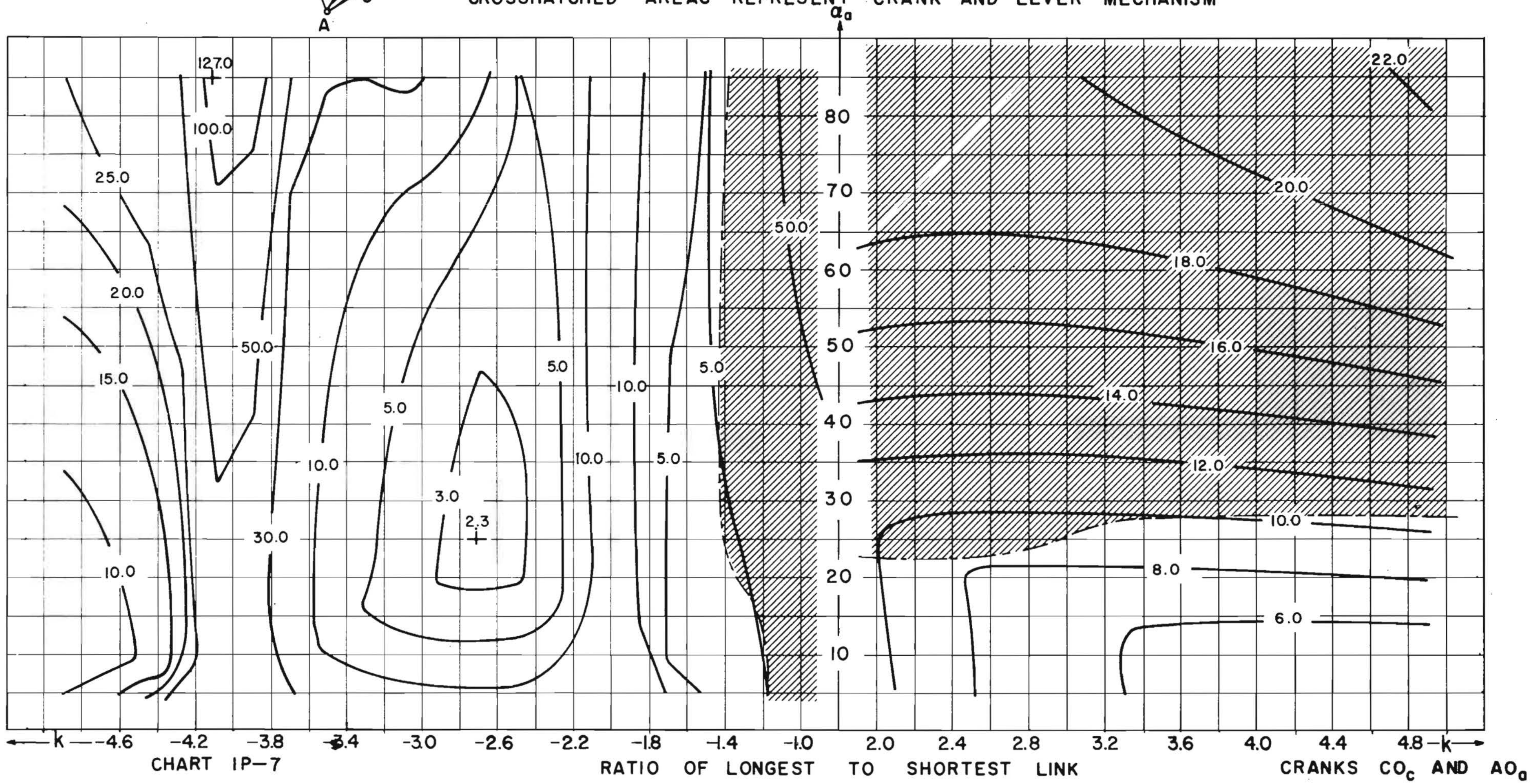
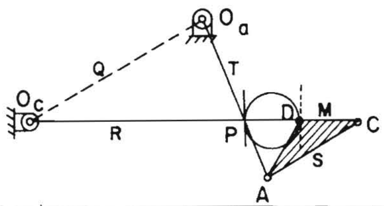


CHART IP-7



BALL-BURMESTER POINT AT INFLECTION POLE

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISM

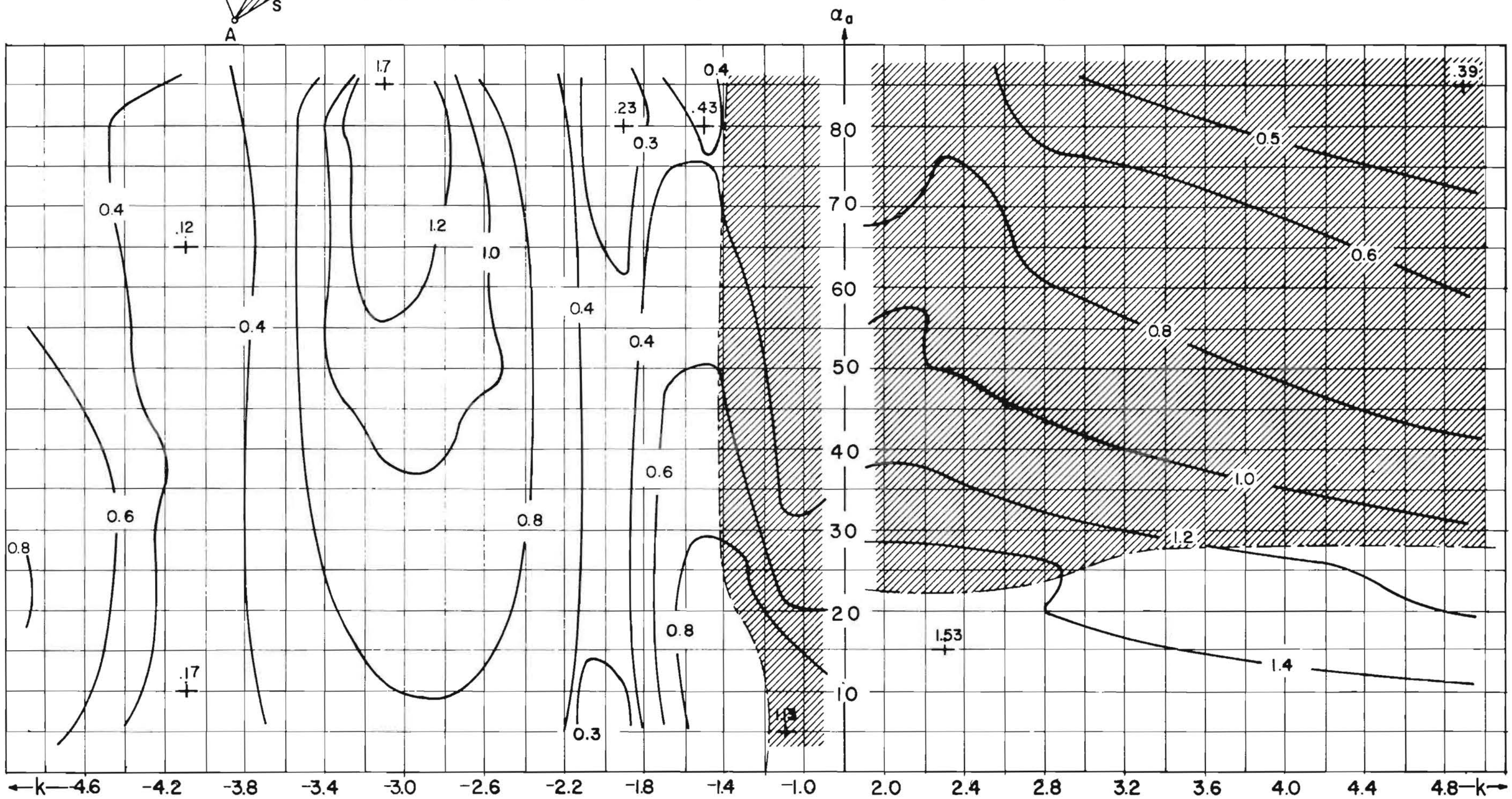
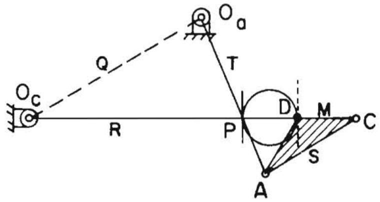


CHART IP-8

STRAIGHT LINE LENGTH L_c, DEVIATION $D_c = 0.01$ UNIT

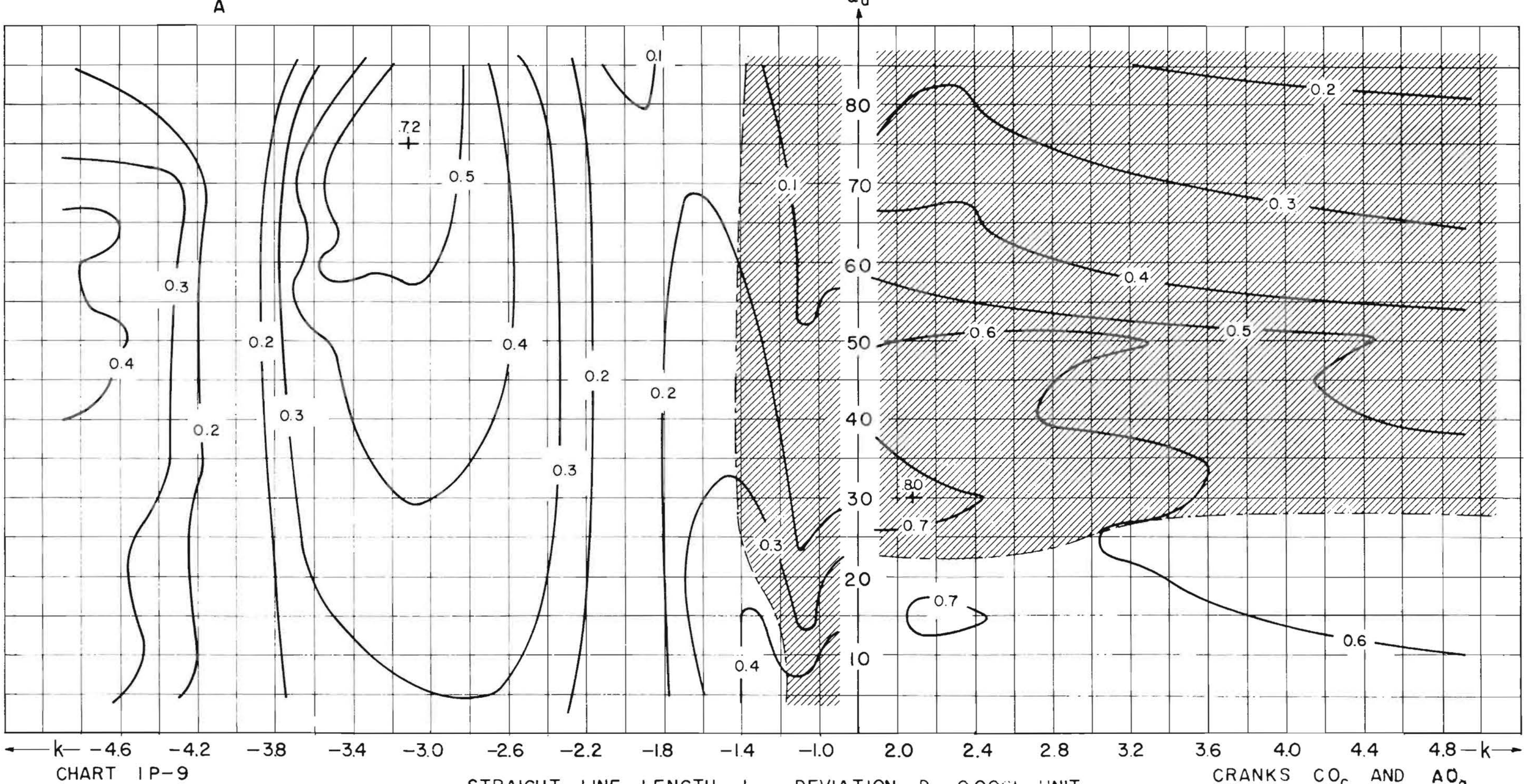
CRANKS CO_c AND AO_a



BALL-BURMESTER POINT AT INFLECTION POLE

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISM

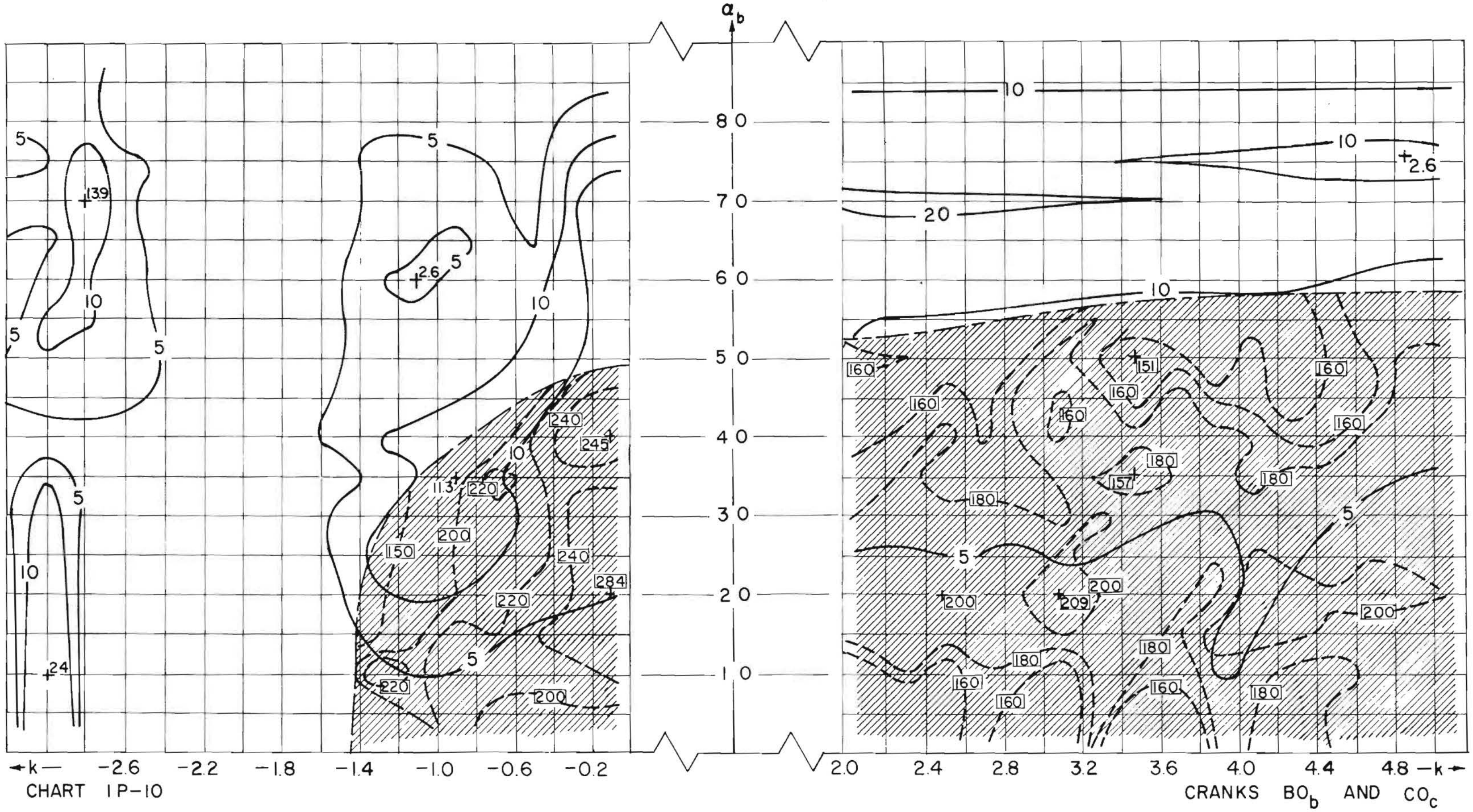


BALL-BURMESTER POINT AT INFLECTION POLE

10 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME MINIMUM TRANSMISSION ANGLE

180 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME CRANK ROTATION ANGLE THESE HAVE NO MEANING FOR A DOUBLE LEVER MECHANISM BECAUSE IT HAS NO CRANK

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISM



MINIMUM TRANSMISSION ANGLES AND CRANK ROTATION ANGLES, DEVIATION D_c = 0.01 UNIT

The Ball-double Burmester Point Case

Chart DB1 contains the ratio of longest to shortest link along with some additional information. Crank-lever linkages are within the cross hatched areas, the rest being double crank types. Due to symmetry of the equation variables, only $\frac{1}{4}$ of the total number of possible linkages are unique and thus plotted. Equations treating the "six-point" and Ball-double Burmester point at inflection pole are given as well.

Charts DB2-DB4 represent straight-line output linkages for accuracies of 0.01, 0.001 and 0.0001. Charts DB5-8 contain crank rotation and transmission angle information.

The coordinates PD for the Ball-double Burmester point are

$$PD = \sin \alpha_d$$

and

$$\alpha_d = \arctan \left\{- \frac{V}{W + 3}\right\}$$

where, as before,

$$V = \tan \alpha_a + \tan \alpha_b$$

$$W = (\tan \alpha_a)(\tan \alpha_b)$$

The coordinates for pin joints A and B are found by

$$PA = \frac{V(W - 1) \sin \alpha_a}{(W + 3) \tan \alpha_a + VW}$$

$$PB = \frac{V(W - 1) \sin \alpha_b}{(W + 3) \tan \alpha_b + VW}$$

The fixed pivot distances are found thus

$$PO_a = - \frac{PA \sin \alpha_a}{PA - \sin \alpha_a}$$

$$PO_b = - \frac{PB \sin \alpha_b}{PB - \sin \alpha_b} \qquad (17)$$

The linkage should be laid out now to check the collinearity of points A, B and D.

The lengths of the cranks can be obtained mathematically by

$$O_aA = \frac{(PA)^2}{PA - \sin \alpha_a}, \qquad O_bB = \frac{(PB)^2}{PB - \sin \alpha_b} \qquad (18)$$

The remaining linkage dimensions are computed by

$$AB = S = \left\{(PA)^2 + (PB)^2 - 2(PA)\,(PB) \cos\,(\alpha_b - \alpha_a)\right\}^{\frac{1}{2}}$$

$$O_aO_b = Q = \left\{(PO_a)^2 + (PO_b)^2 - 2(PO_a)\,(PO_b) \cos\,(\alpha_b - \alpha_a)\right\}^{\frac{1}{2}} \qquad (19)$$

$$AD = M = \left\{(PA)^2 + (PD)^2 - 2(PA)\,(PD) \cos\,(\alpha_d - \alpha_a)\right\}^{\frac{1}{2}}$$

When the choice of parameters is such that

$$\alpha_a = \alpha_b + 60$$

contact with six infinitesimally-separated points (fifth-order) occurs (proven by R. Mueller, 1901).

If the Ball-double Burmester point lies at the inflection pole, considerable simplification of the equations occurs.

$$\alpha_d = 90°, \; PD = PJ = 1.0$$

$$\alpha_b = \arctan \left\{- \frac{3.0}{\tan \alpha_a}\right\} \qquad (20)$$

$$PA = (4/3) \sin \alpha_a, \qquad PO_a = -3(PA)$$

$$PB = (4/3) \sin \alpha_b, \qquad PO_b = -3(PB)$$

Example 1.

Because of the relative uniqueness of this case, several examples are considered. The first is given with numerical calculations. Choosing α_a = 10.0° and α_b = 115.0° results in a linkage of good proportions (Chart DB1), satisfactory straight-line output (Charts DB2-DB4), a particularly large rotation angle for the input crank (Charts DB5-DB6), and a very satisfactory minimum value for the transmission angle at the follower pin (Charts DB7-DB8).

$$V = 0.176 - 2.145 = -1.969$$

$$W = (0.176)\ (-2.145) = -0.377$$

then by Eqs. (15)

$$\alpha_d = \arctan\left\{-\frac{-1.969}{-0.377 + 3.0}\right\} = 36.9°$$

and

$$PD = \sin\ (36.9°) = 0.600''$$

The location of the pin joints is obtained by Eqs. (16)

$$PA = \frac{(-1.969)\ (-1.377)\ (0.1737)}{(-0.377 + 3.0)\ (0.176) + (-0.377)\ (-1.969)} = 0.391''$$

$$PB = \frac{(-1.969)\ (-1.377)\ (0.907)}{(-0.377 + 3.0)\ (-2.145) + (-0.377)\ (-1.969)} = -0.503''$$

and the location of the fixed pivots by Eqs. (17)

$$PO_a = \frac{-(0.391)\ (0.1737)}{(0.391) - (0.1737)} = -0.313''$$

$$PO_b = \frac{-(-0.503)\ (0.907)}{(-0.503) - (0.907)} = -0.323''$$

As recommended, the linkage should now be laid out accurately. If points A, B and D are collinear, the computation is probably correct. The full range of the motion can now be studied by the designer. The value for the length L_c of the straight-line output is read from Chart DB2 to be 1.36. Since the linkage as computed above is given at a different scale, L_c must be adjusted. To accomplish this, the unit number UN is computed. An accurate value for UN requires the use of Eqs. (18) and (19) with Eq. (3). The authors recommend, however, that these dimensions be scaled from the drawing and used as follows

$$UN = \frac{Q + R + S + T + \frac{M + N}{2}}{5}$$

$$\approx \frac{0.505 + 0.703 + 0.55 + 0.18 + \frac{0.86 + 0.31}{2}}{5} = 0.504$$

which is very close to the value given by digital computation. Then the actual length is

$$L_a = (L_c)\ (UN) = (1.36)\ (0.504) = 0.686''$$

but now the accuracy becomes

$$D_a = (D_c)\ (UN) = (0.01)\ (0.504) = 0.005''$$

(If each of the linkage dimensions is divided by the above value of UN, then the dimensions given in Figure 9 would result).

Suppose it is desired to have a straight-line output 5 inches long and accurate within that length to 0.001 inch. By considering the chart we note that (L_c) 0.0001 = 0.48. To adjust this length to agree with the graphical layout.

PARAMETERS

$\alpha_a = 10.0 \qquad \alpha_b = 115.0$

DIMENSIONS (α_d = 36.895)

Q =	1.0007	S =	1.0932
R =	1.3930	M =	0.6099
T =	0.3567	ϵ =	180.0

RESULTS (ϕ_o = 38.264)

D	0.0001	0.001	0.01
L	0.480	0.776	1.363
$\Delta\psi$	65.1	107.2	218.3
γ_b	24.2	11.8	11.8

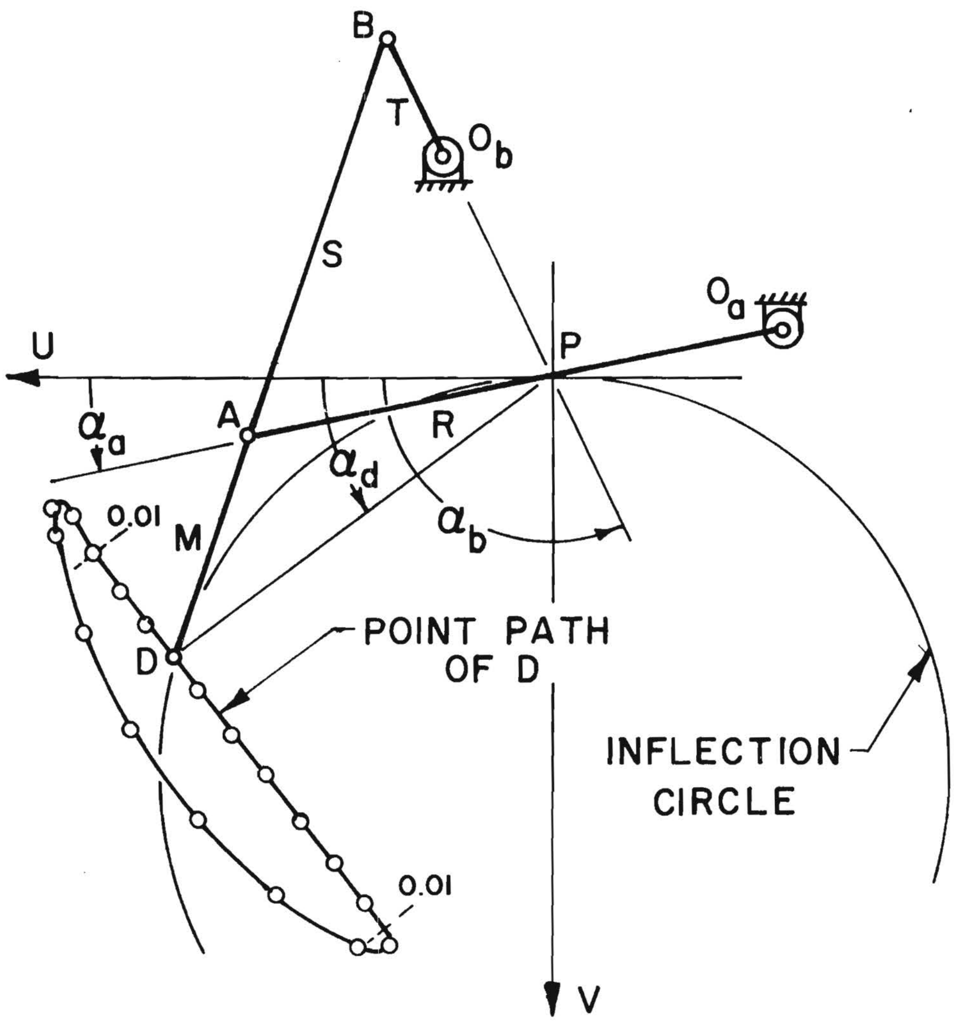


Figure 9. Ball-double Burmester Linkage.

$$L_a = (0.48)\ (0.504) = 0.242''$$

but

$$D_a = (0.0001)\ (0.504) = 0.0000504''$$

To obtain the desired length, the enlargement factor by which the mechanism must be changed is

$$E.F. = \frac{L_d}{L_a} = \frac{5.0}{.242} = 20.65$$

If each dimension is changed by this factor, then

$$UN = (0.504)\ (20.65) = 10.4$$

and the accuracy becomes

$$D_a = (10.4)\ (0.0001) = 0.00104''$$

Hence, it follows that with further enlargement a line four feet long and accurate to 0.01 inch can be produced by this linkage. Linkages designed by methods not based on curvature theory would not, in general, give such a long and accurate straight-line.

Example 2. Long output for accuracy of 0.01.

On Chart DB2, the parameter values $\alpha_a = 15.0°$ and $\alpha_b = 20.0°$ indicate that a value of $L_c = 1.73$ occurs for an accuracy of 0.01. Charts DB3 and DB4 also show that good results can be expected. The linkage is represented in Figure 10 with its dimensions adjusted so that UN = 1.0. Disregarding the length of the coupler BAD, note the relatively long straight-line output in comparison to the size of the working elements, Q, R, S, and T.

PARAMETERS

$\alpha_a = 15.0 \qquad \alpha_b = 20.0$

DIMENSIONS (α_d = 168.469)

Q =	0.2782	S =	0.2971
R =	1.3275	M =	1.8556
T =	1.0930	ϵ =	180.0

RESULTS (ϕ_o = 314.450)

D	0.0001	0.001	0.01
L	0.7303	1.1294	1.7318
$\Delta\psi$	31.34	48.34	74.15
γ_b	72.3	66.3	53.7

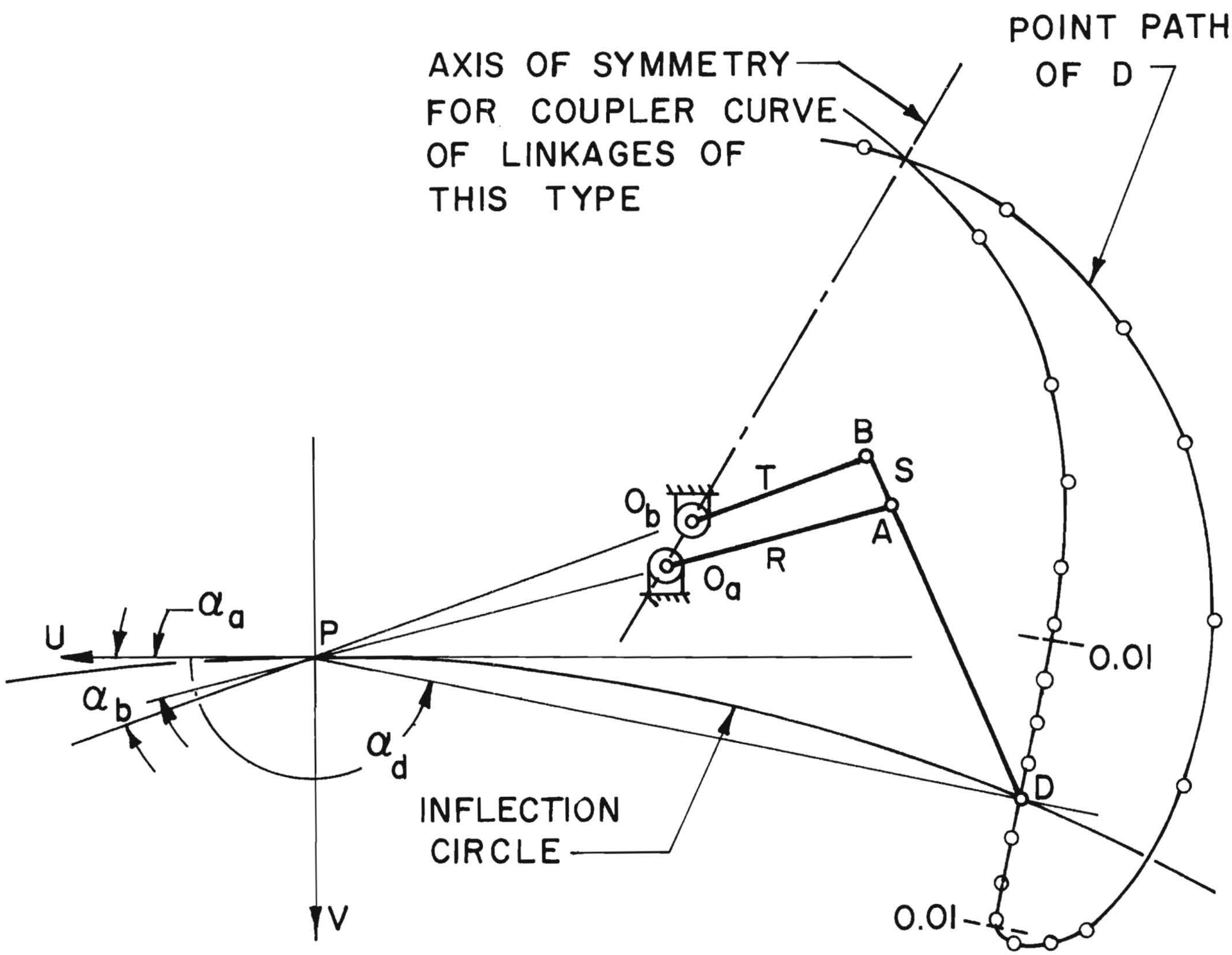


Figure 10. Long Output Linkage.

Example 3. Long output for accuracy of 0.0001.

By choosing the parameter values $\alpha_a = 5.0°$ and $\alpha_b = 60.0°$, Chart DB4 indicates that a particularly large value of $L_c = 0.79$ results. In addition, the linkage is a crank-and-lever mechanism but has a ratio (Chart DB1) of longest to shortest link of 8.9. Charts DB5-DB8 indicate very useful values for the crank rotation angle and follower transmission angle. As shown in Figure 11, the working elements are quite compact and removed from the vicinity of the output point. One of the difficulties of providing straight-line motion by slides on ways for high speed machinery is the necessary concentration of mass where high acceleration peaks occur. With this linkage, the slower moving working elements can be built for strength and the extension AD of the coupler link made quite light to reduce inertia forces.

Example 4. Ball-double Burmester point at the inflection pole.

Using the design value $\alpha_a = 40.0°$ with Eqs. (20), the linkage in Figure 12 results. The Roberts-Chebishev theory permits two other linkages which are represented on the figure and which generate the same coupler curve. Many times these cognate linkages will prove more useful to designers than the original.

Example 5. A "six point exact" linkage.

By choosing $\alpha_b = \alpha_a + 60$, there results a special linkage having fifth order contact with a straight-line. This case has triangle PAB an equilateral triangle (Fig. 13).

PARAMETERS

$\alpha_a = 5.0 \qquad \alpha_b = 60.0$

DIMENSIONS (α_d = 150.000)

$Q = 0.7449 \qquad S = 1.0530$

$R = 0.8584 \qquad M = 1.5812$

$T = 0.2361 \qquad \epsilon = 180.0$

RESULTS (ϕ_o = 285.293)

D	0.0001	0.001	0.01
L	0.791	1.263	1.405
$\Delta\psi$	91.1	161.4	187.5
γ_b	22.6	15.6	15.6

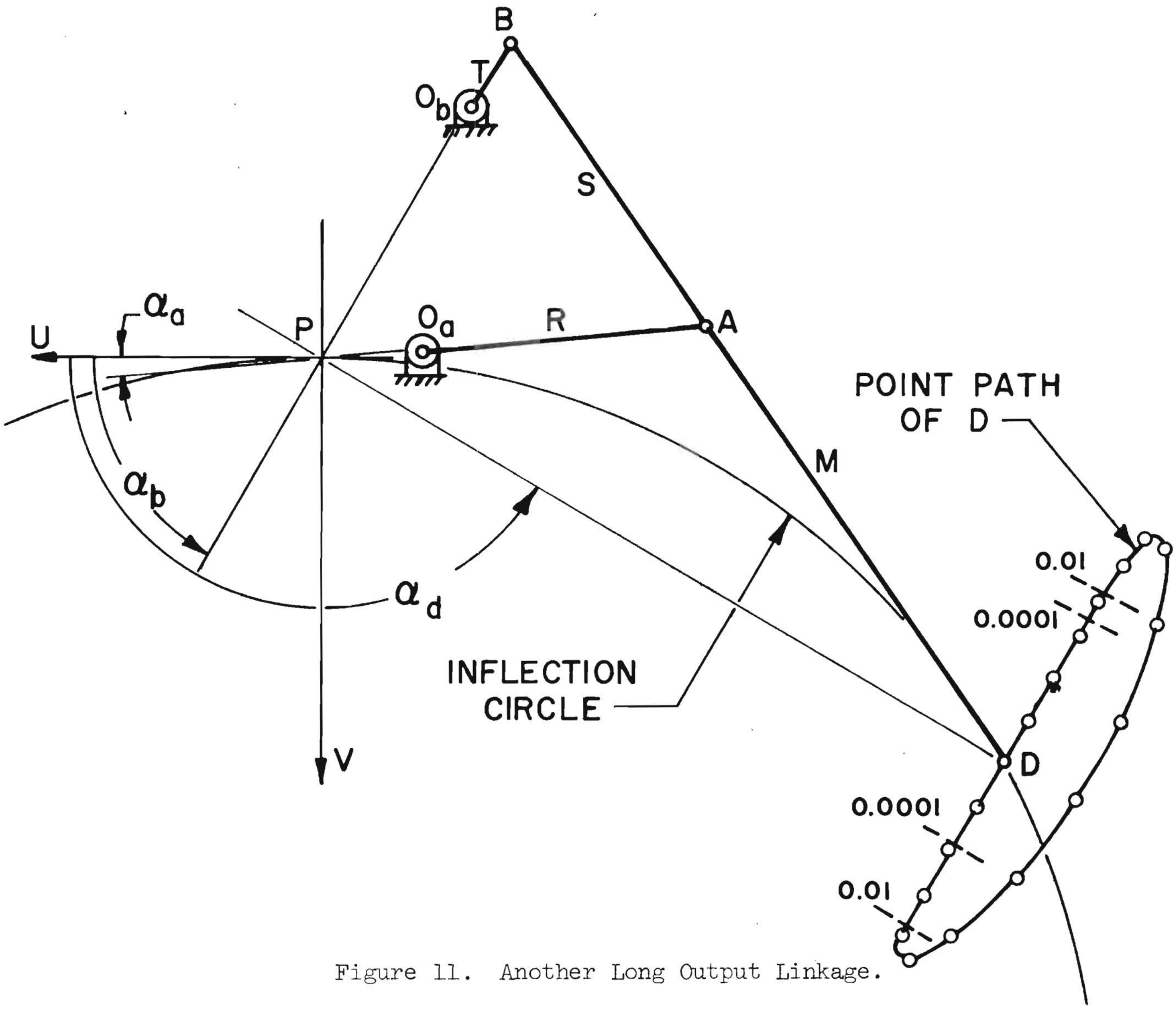


Figure 11. Another Long Output Linkage.

PARAMETERS

$\alpha_a = 40.0$

DIMENSIONS (α_d = 90.000 , α_b = 105.65)

Q =	1.2985	S =	0.4328
R =	1.2218	M =	0.2835
T =	1.8305	ϵ =	0.0

RESULTS (ϕ_o= 74.373)

D	0.0001	0.001	0.01
L	0.420	0.692	1.814
$\Delta\psi$	16.7	27.4	36.2
γ_b	15.3	4.4	0.4

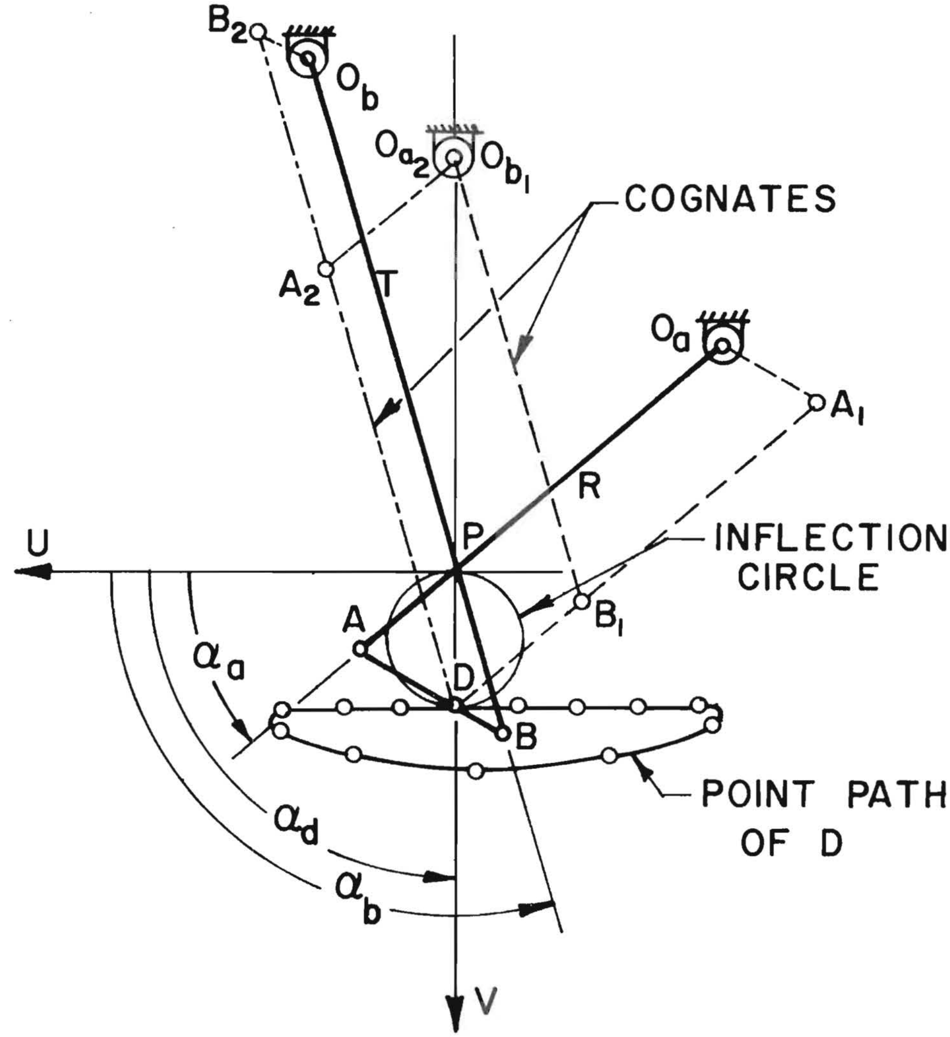


Figure 12. Cognate Linkages.

PARAMETERS

$\alpha_a = 55.0$ $\alpha_b = 115.0$

DIMENSIONS ($\alpha_d = 95.000$)

Q	=	1.2906	S	=	0.3668
R	=	1.3247	M	=	0.2394
T	=	1.8344	ϵ	=	0.0

RESULTS ($\phi_o = 80.000$)

D	0.0001	0.001	0.01
L	0.509	0.796	0.908
$\Delta\psi$	18.1	29.4	31.4
γ_b	21.9	14.2	1.1

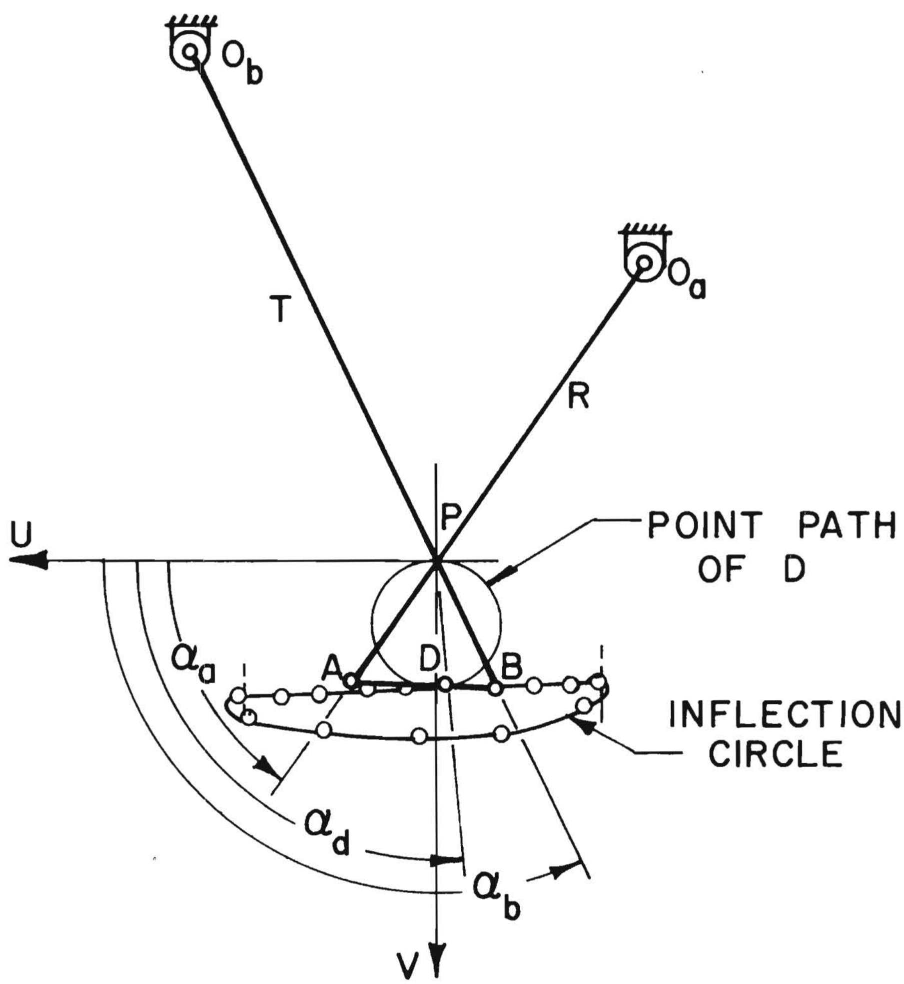


Figure 13. "Six Point" Linkage.

BALL-DOUBLE BURMESTER POINT

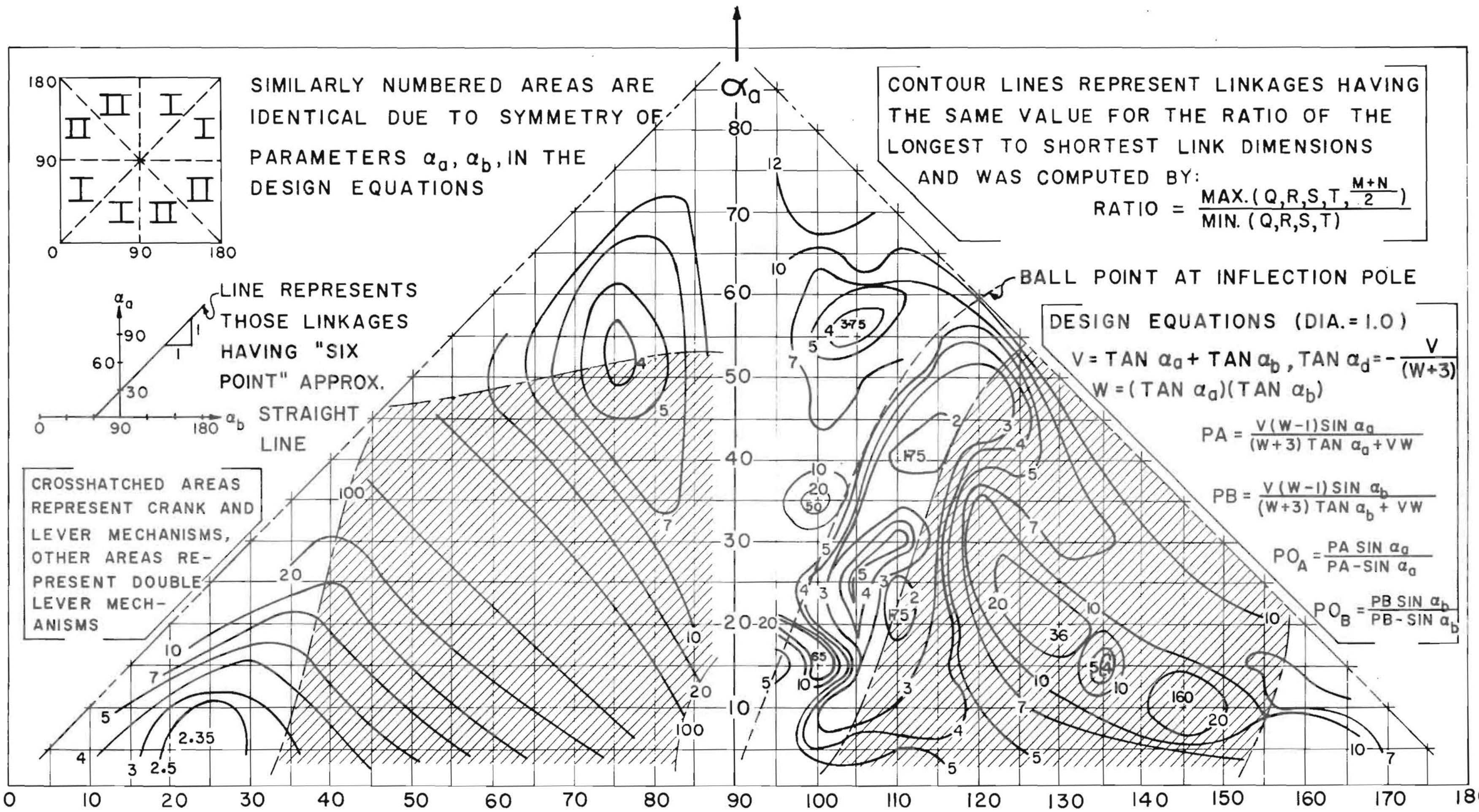


CHART DB I RATIO OF LONGEST TO SHORTEST LINK $\alpha_b \longrightarrow$

BALL-DOUBLE BURMESTER POINT

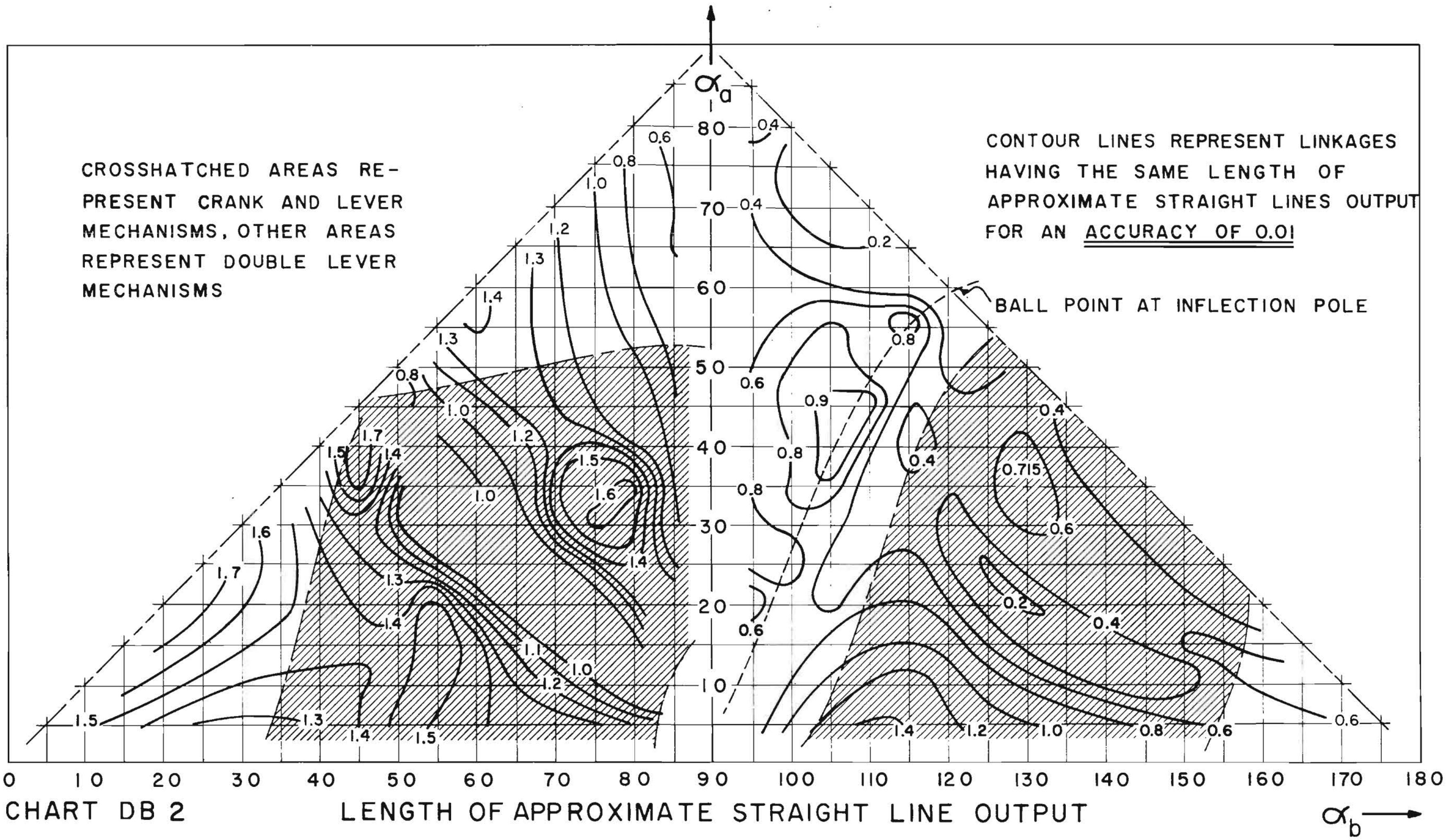


CHART DB 2

BALL-DOUBLE BURMESTER POINT

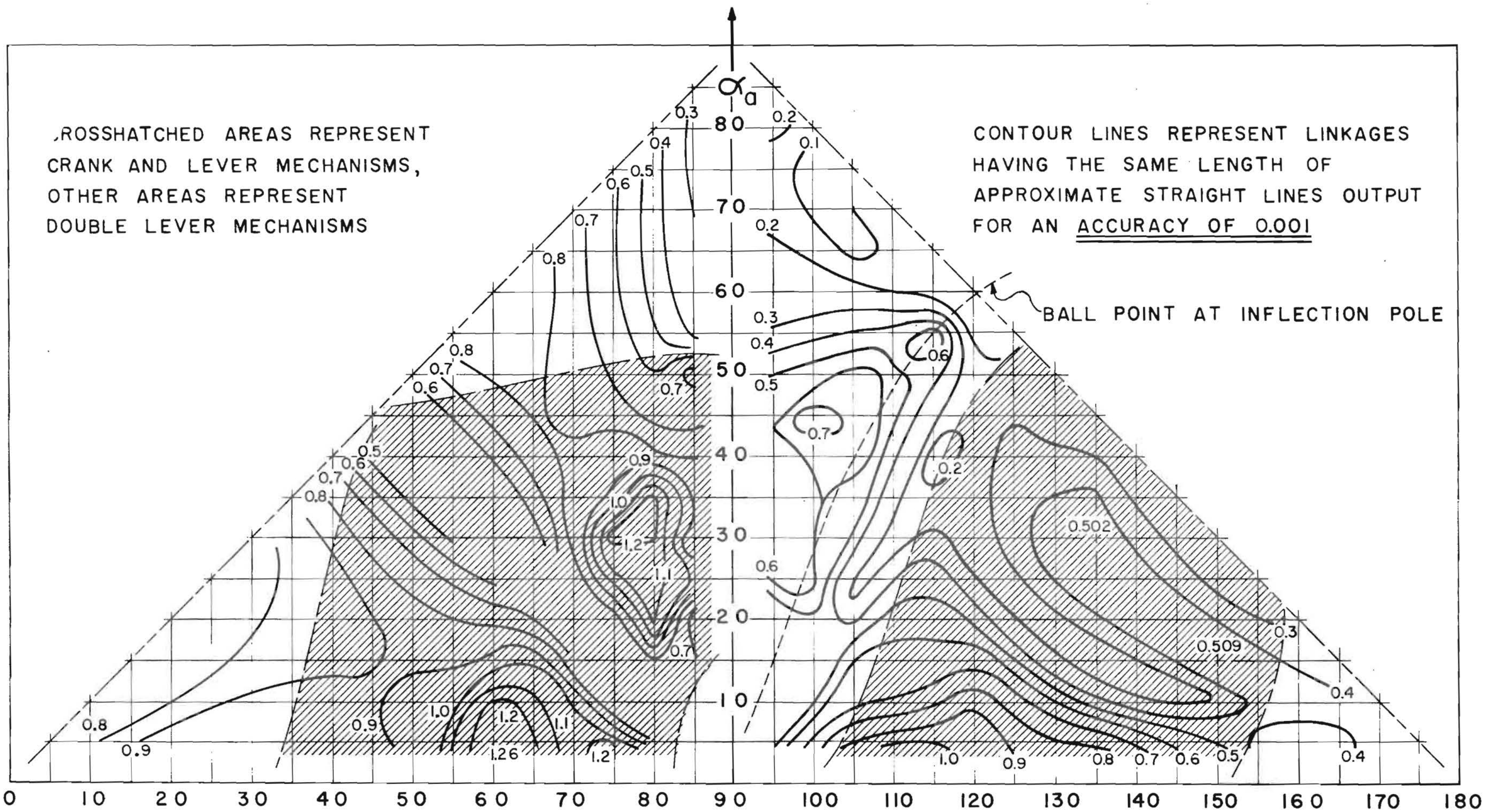


CHART DB 3 LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT $\alpha_b \rightarrow$

BALL-DOUBLE BURMESTER POINT

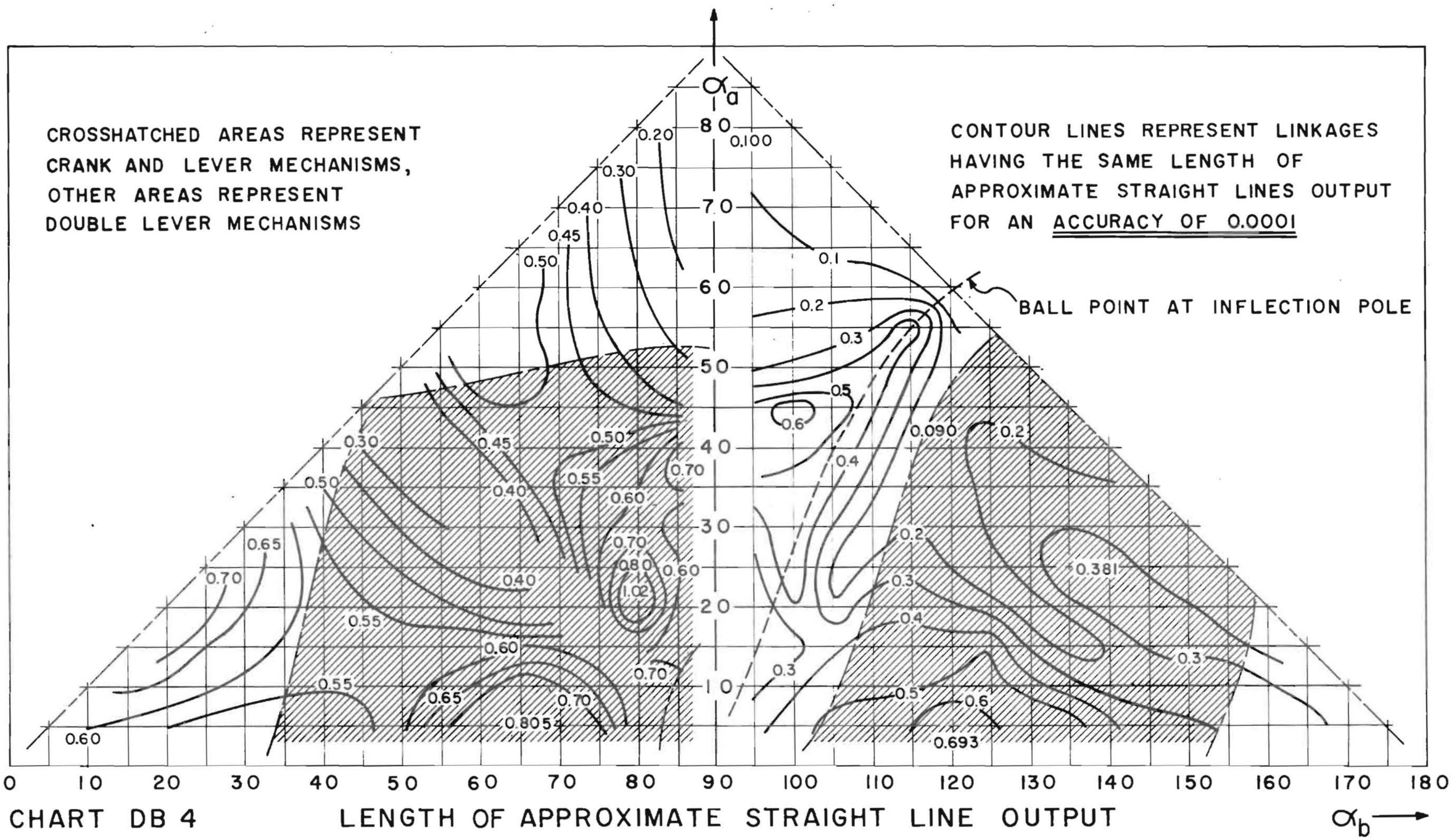


CHART DB 4

BALL-DOUBLE BURMESTER POINT

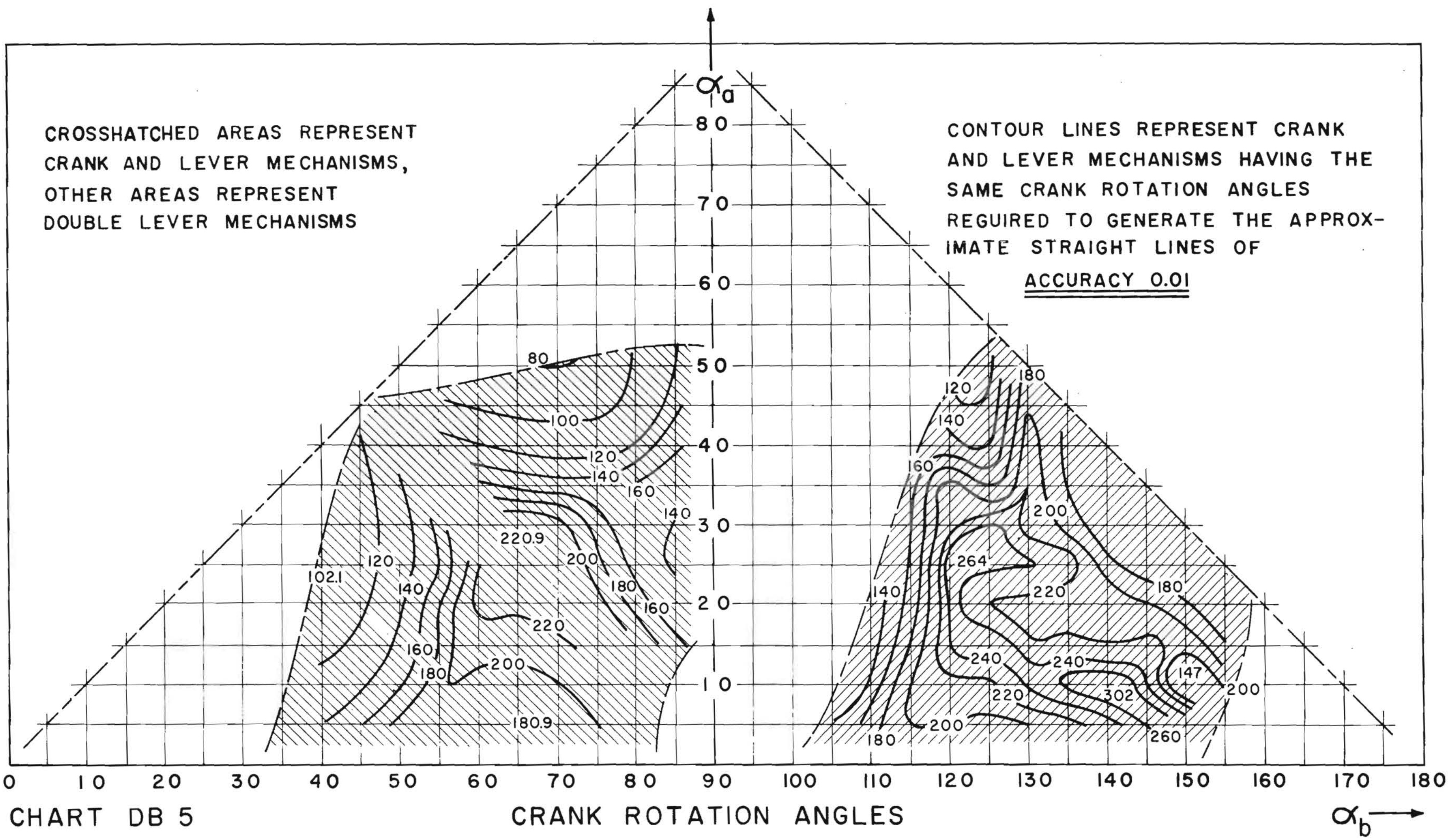


CHART DB 5

BALL-DOUBLE BURMESTER POINT

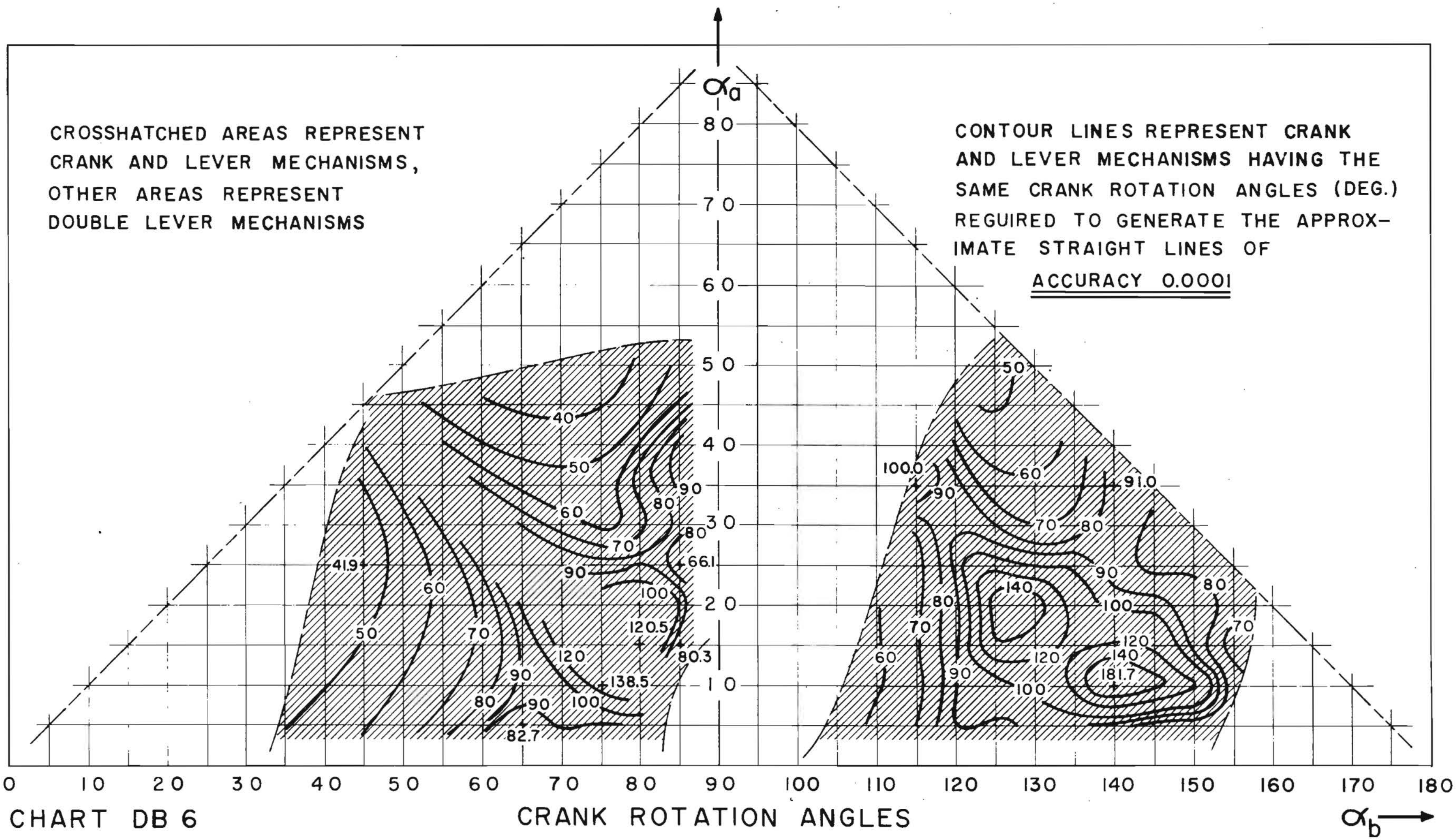


CHART DB 6

BALL-DOUBLE BURMESTER POINT

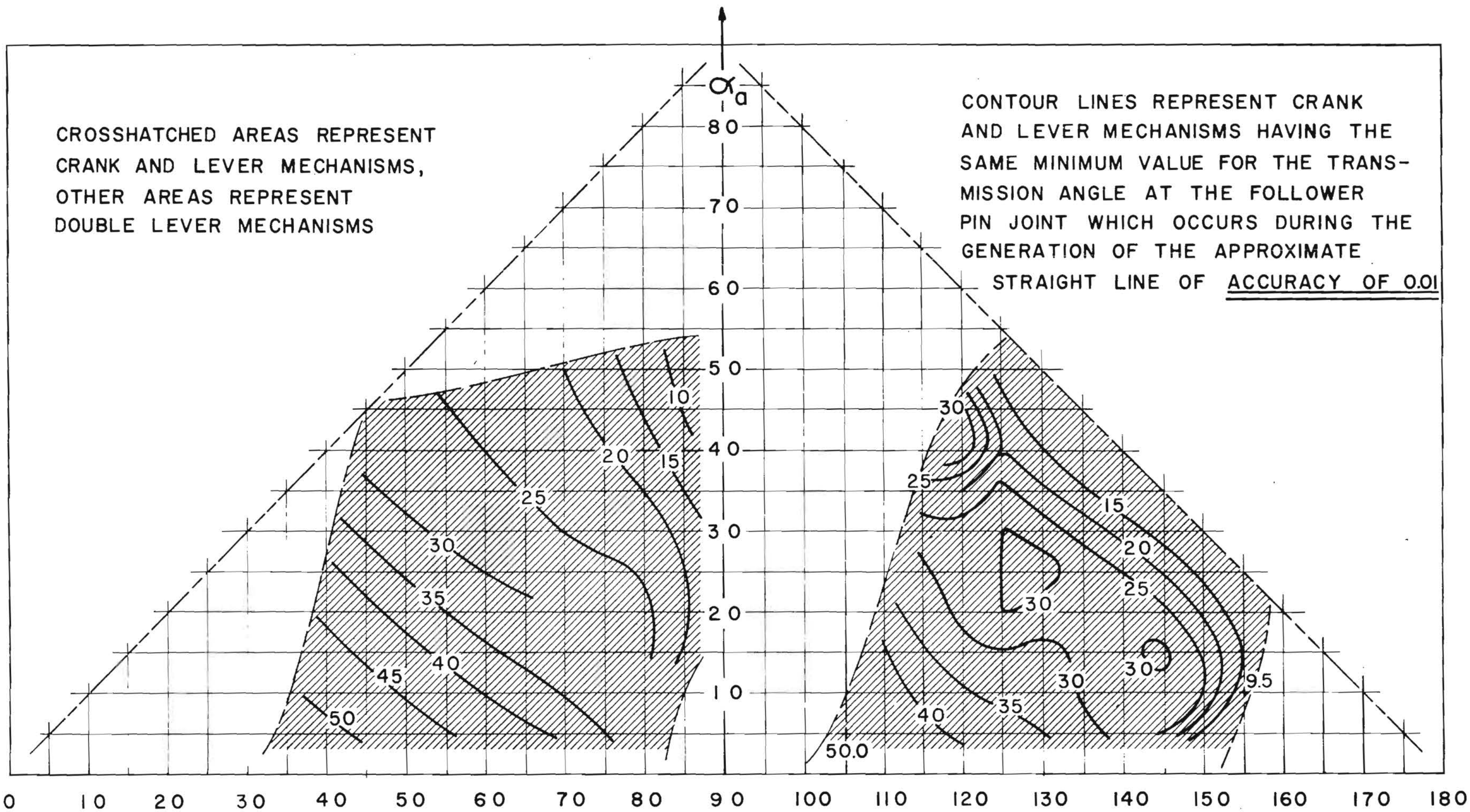
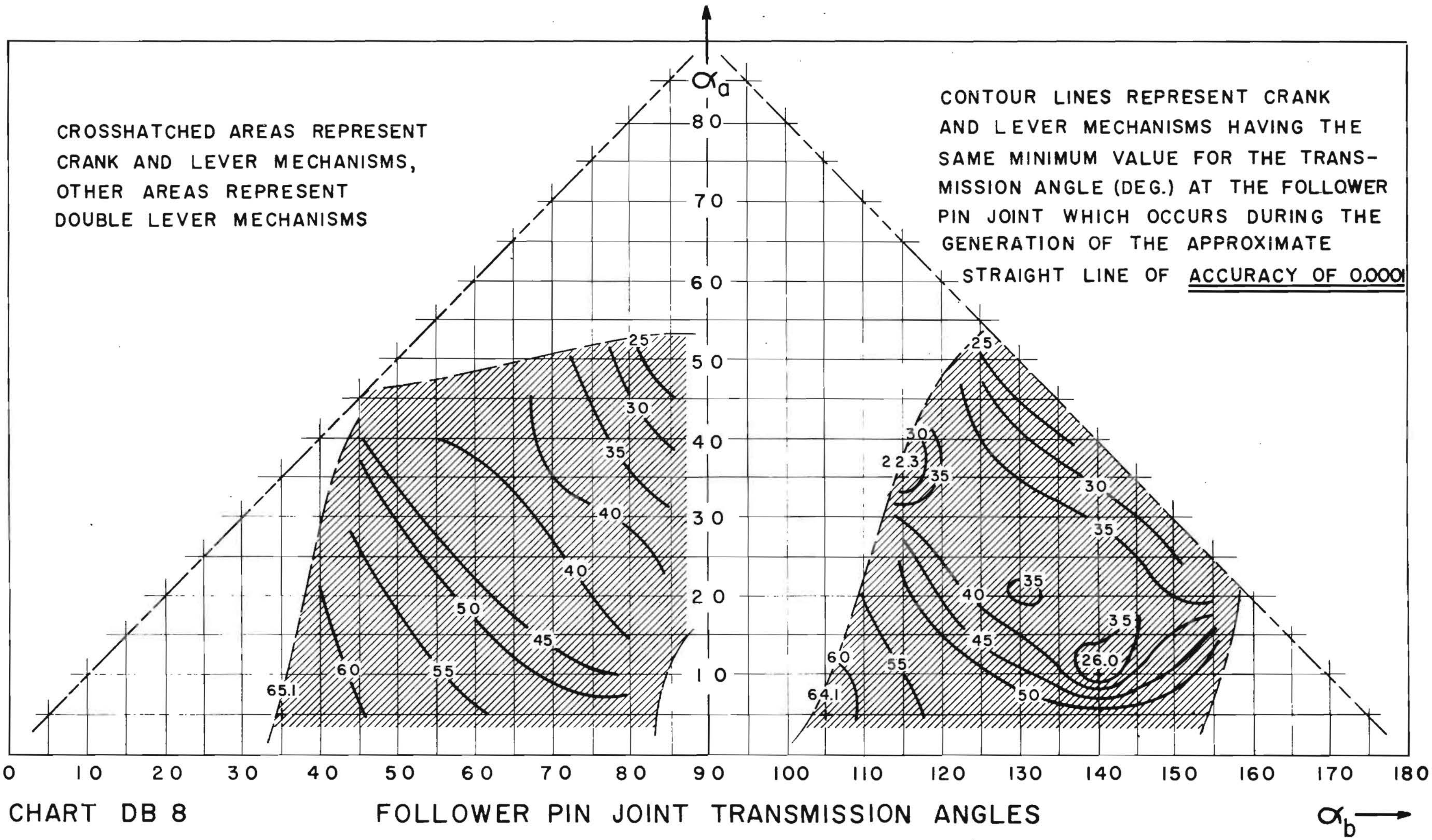


CHART DB 7

FOLLOWER PIN JOINT TRANSMISSION ANGLES

BALL-DOUBLE BURMESTER POINT



The Watt Straight-Line Mechanism Case

The Watt linkage is, of course, a straight-line mechanism that has been of great importance for a long time. Charts W1 to W4 and W6 present the straight-line outputs at a deviation of 0.01 and for coupler inclinations from 30° to 90° in intervals of 15°. Charts W5 and W7 contain similar data for a deviation of 0.001 but only for the 75° and 90° inclinations. Note also that the parameter η specifies an open mechanism when negative and a crossed when positive.

Link lengths are identified by the same letters previously given and shown in Fig. 14. The coupler dimension M and N, also shown in the figure, are determined as follows.

$$M = ST/(T + \eta R) \tag{21}$$

$$N = \eta(S - M) \tag{22}$$

and the fixed link length is

$$Q = S^2 + (T - R)^2 - 2S(T - R) \cos \gamma \tag{23}$$

Example

An example will illustrate the use of the charts. Suppose the designer chooses:

$$S = 1.0,\ R = 1.0,\ T = 2.0,\ \gamma = 75.0^\circ,\ \eta = -1$$

then the data

$$D_c = 0.01,\ \ L_c = 1.79$$

$$D_c = 0.001,\ L_c = 0.430$$

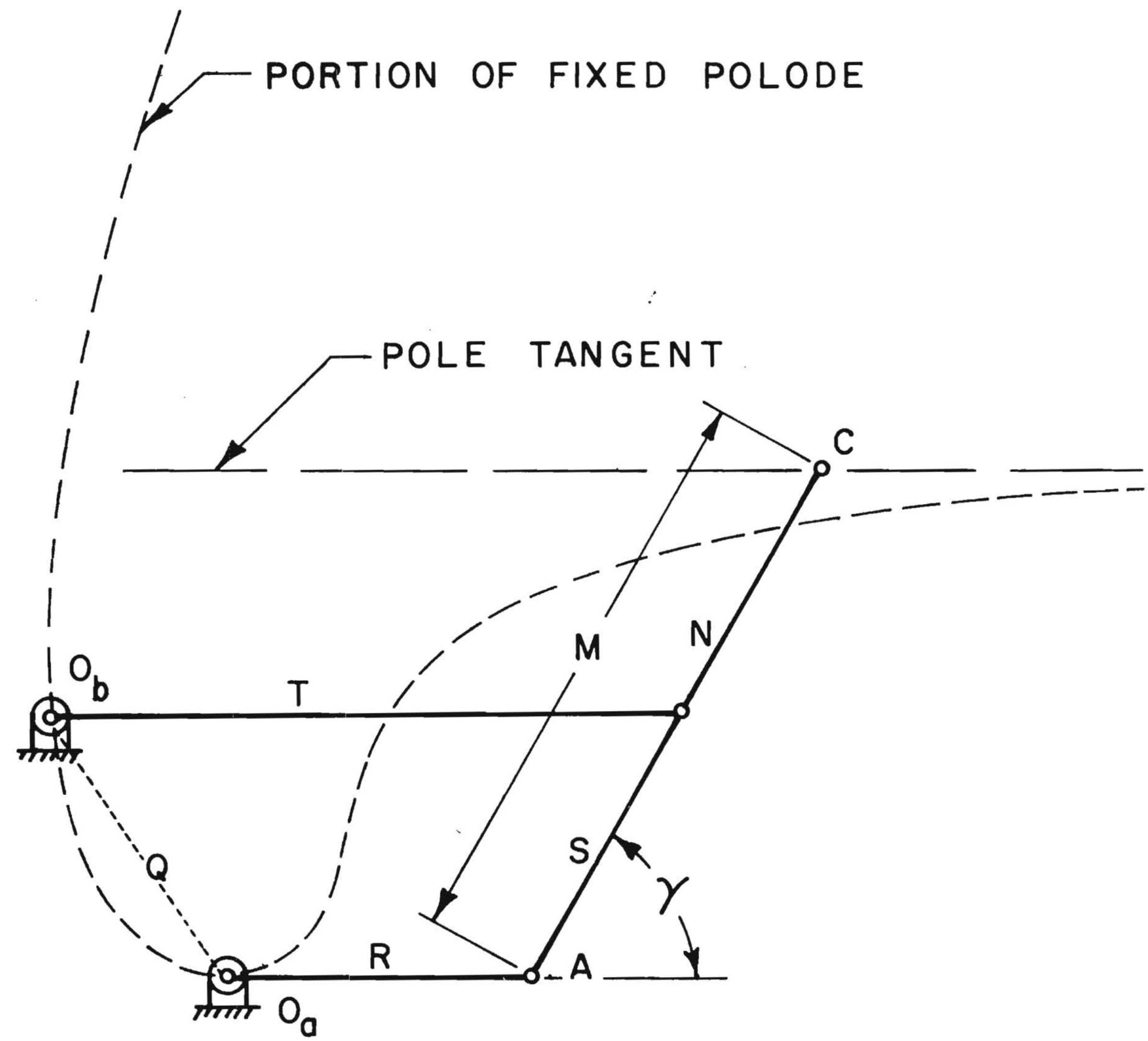


Figure 14. General Form of Watt Mechanism.

are obtained from charts W4 and W5, respectively. The other dimensions are obtained using Eqs. (21) thru (23),

$$M = \frac{ST}{T + \eta R} = \frac{(1.0)(2.0)}{(2.0) - 1 \times (1.0)} = 2.00$$

$$N = \eta(S - M) = -1 \times (1.0 - 2.00 = 1.00$$

$$Q = \sqrt{S^2 + (T - R)^2 - 2S(T - R) \cos \gamma}$$

$$= \sqrt{(1.0)^2 + (2.0 - 1.0)^2 - 2 \times (1.0) \cos 75^\circ}$$

$$= 1.21$$

and by application of Eq. (3), the value of UN_a is calculated as

$$UN_a = \frac{Q + R + S + T + \frac{M + N}{2}}{5}$$

$$= \frac{1.21 + 1.0 + 1.0 + 2.0 + \frac{2.0 + 1.0}{2.0}}{5.0}$$

$$= 1.34$$

Thus for the first set of chart values

$$L_a = L_c(UN_a) = 1.79 \times 1.34 = 2.39''$$

$$D_a = D_c(UN_a) = 0.01 \times 1.34 = 0.013''$$

and for the second set

$$L_a = L_c(UN_a) = 0.430 \times 1.34 = 0.576''$$

$$D_a = D_c(UN_a) = 0.001 \times 1.34 = 0.0013''$$

The mechanism is represented in Figure 15.

Suppose a mechanism having an overall average size of 10" is desired. Then each of the dimensions R, S, T, etc. must be increased by the factor $UN = \frac{10.0}{UN_a}$ and the L_c and D_c values must be increased by a factor of 10. In this way the mechanism would produce a straight line output with the accuracy of 0.1" for 23.9" or 0.01" for 5.76". Another increase in the magnitude of the linkage by a factor of ten would provide a straight line output 20 ft. long for an accuracy of 1" or about 5 ft. long for an accuracy of 0.1".

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PARAMETERS

$\eta = -1 \quad \gamma = 75^\circ \quad R = 1.0 \quad T = 2.0$

DIMENSIONS

$Q = 0.9062 \qquad S = 0.7443$

$R = 0.7443 \qquad M = 1.4886$

$T = 1.4886 \qquad \epsilon = 0.0$

RESULTS

D	0.0001	0.001	0.01
L	0.195	0.427	1.793

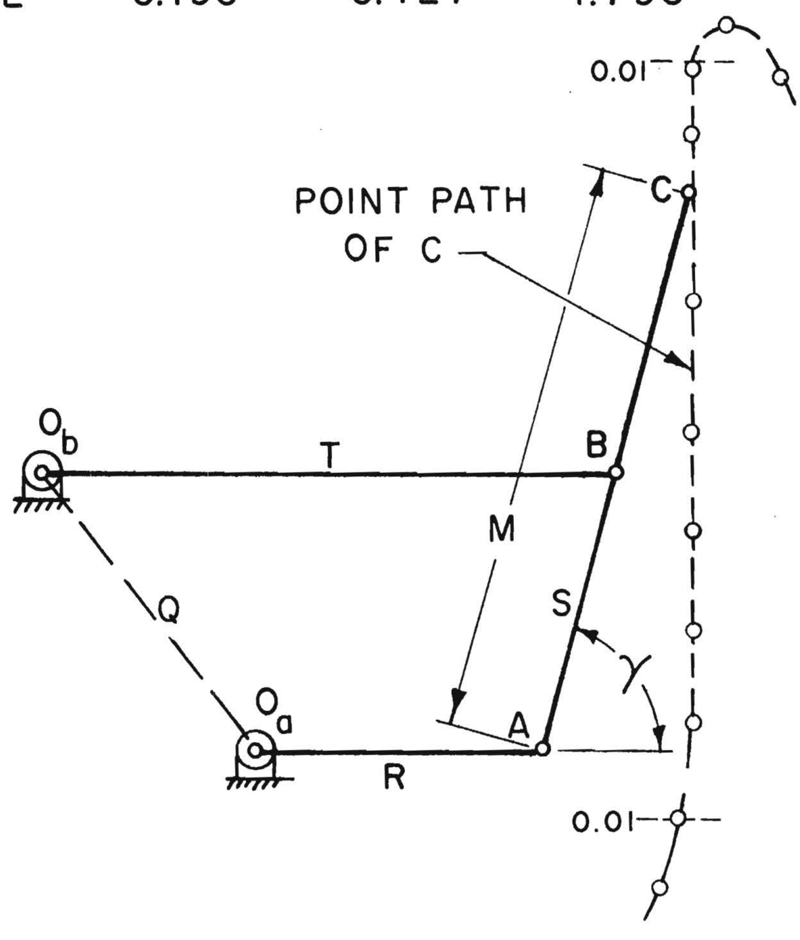
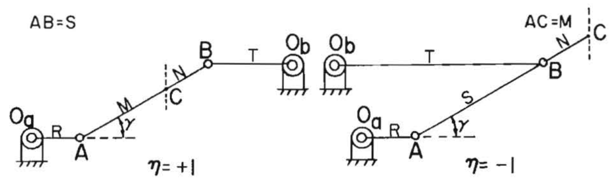


Figure 15. Example, Watt Mechanism.

WATT MECHANISM



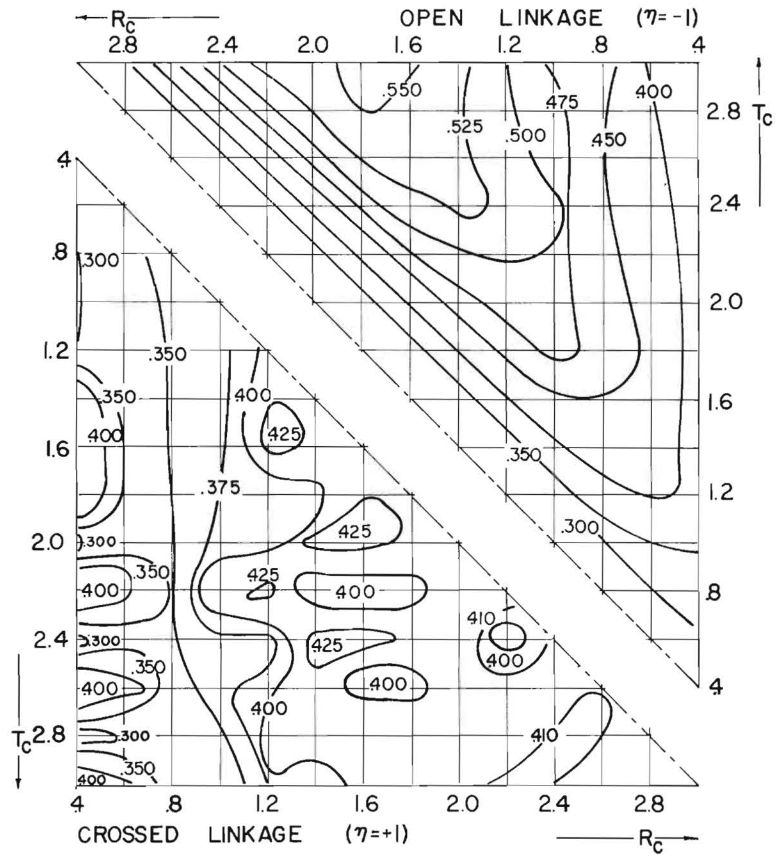
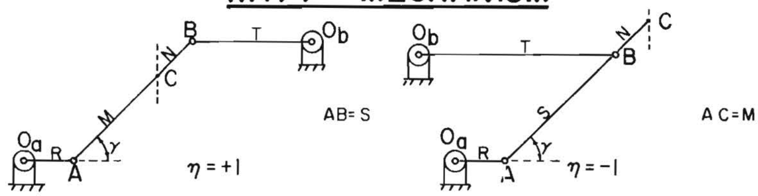


CHART WI, STRAIGHT LINE LENGTH L_c
DEVIATION D_c=0.01 UNIT
GAMMA γ=30°

WATT MECHANISM



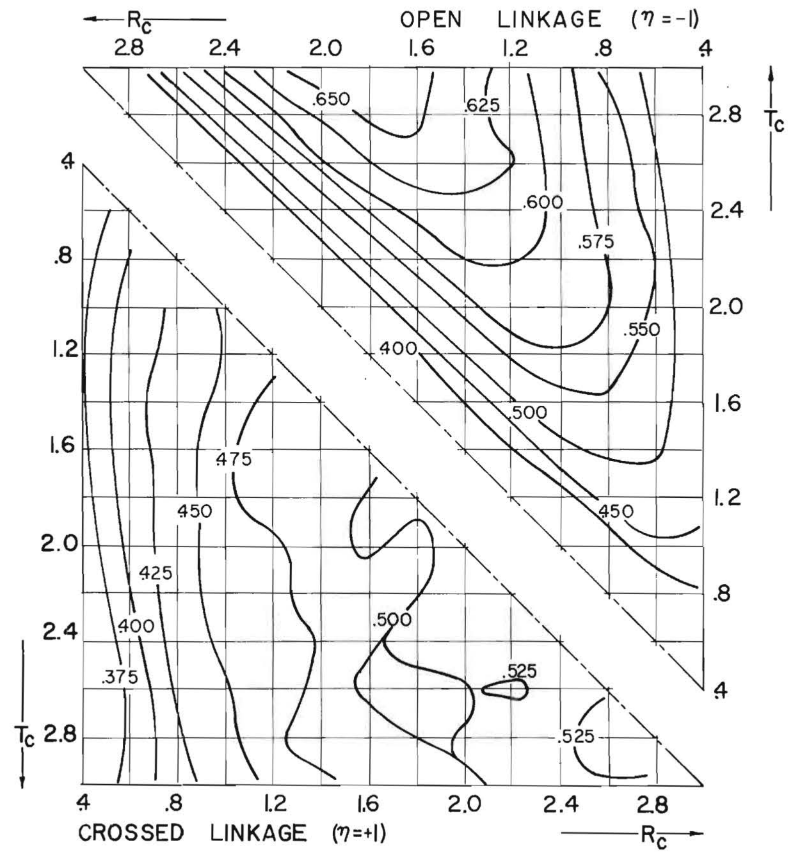


CHART W2, STRAIGHT LINE LENGTH L_c
DEVIATION D_c=0.01 UNIT
GAMMA γ=45°

WATT MECHANISM

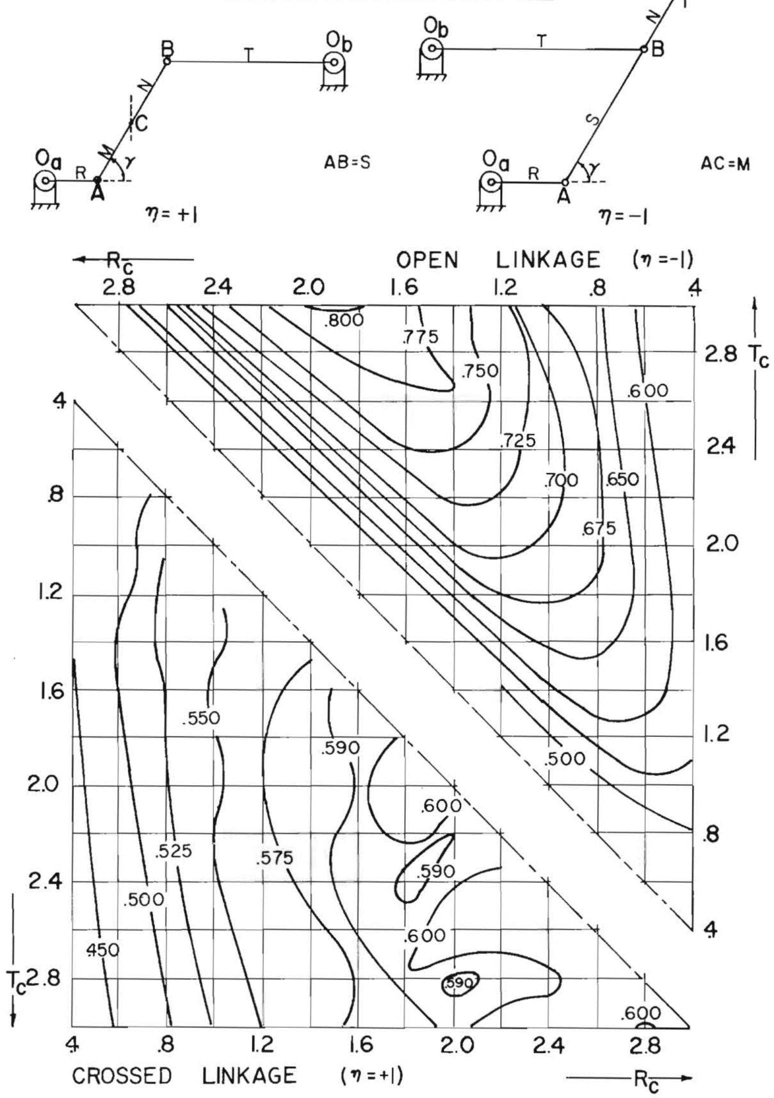


CHART W3, STRAIGHT LINE LENGTH L_c
DEVIATION D_c=0.01 UNIT
GAMMA γ=60°

WATT MECHANISM

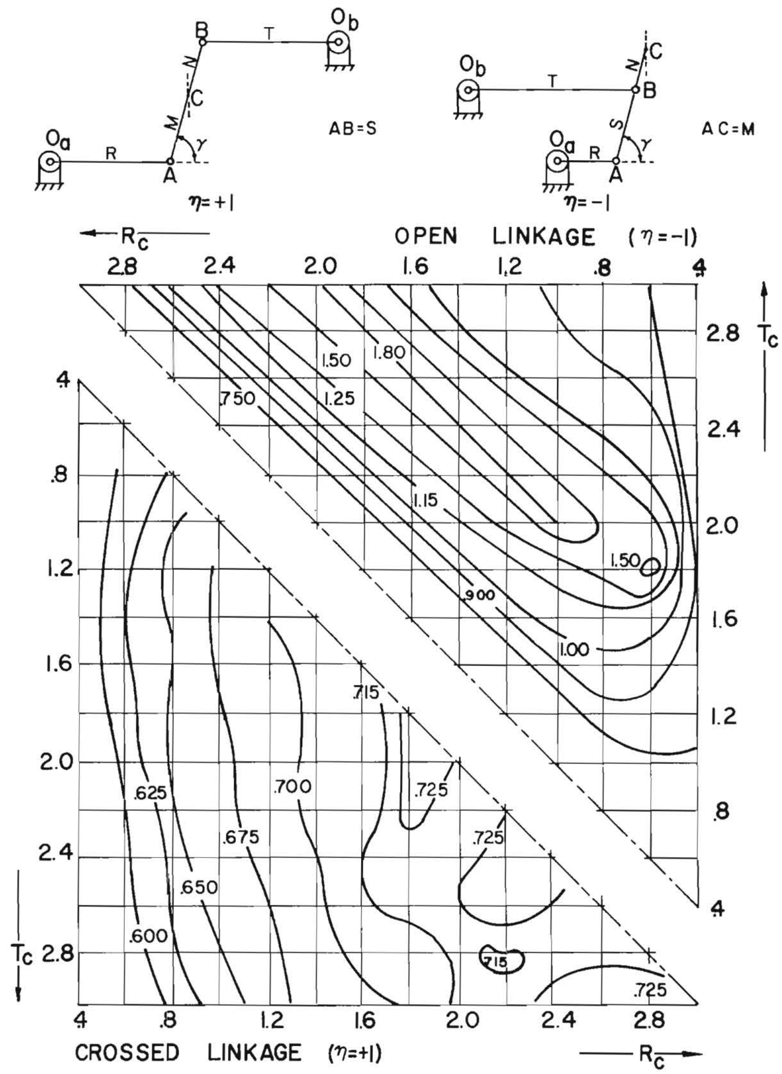


CHART W4, STRAIGHT LINE LENGTH L_c

DEVIATION D_c=0.01 UNIT

GAMMA γ=75°

WATT MECHANISM

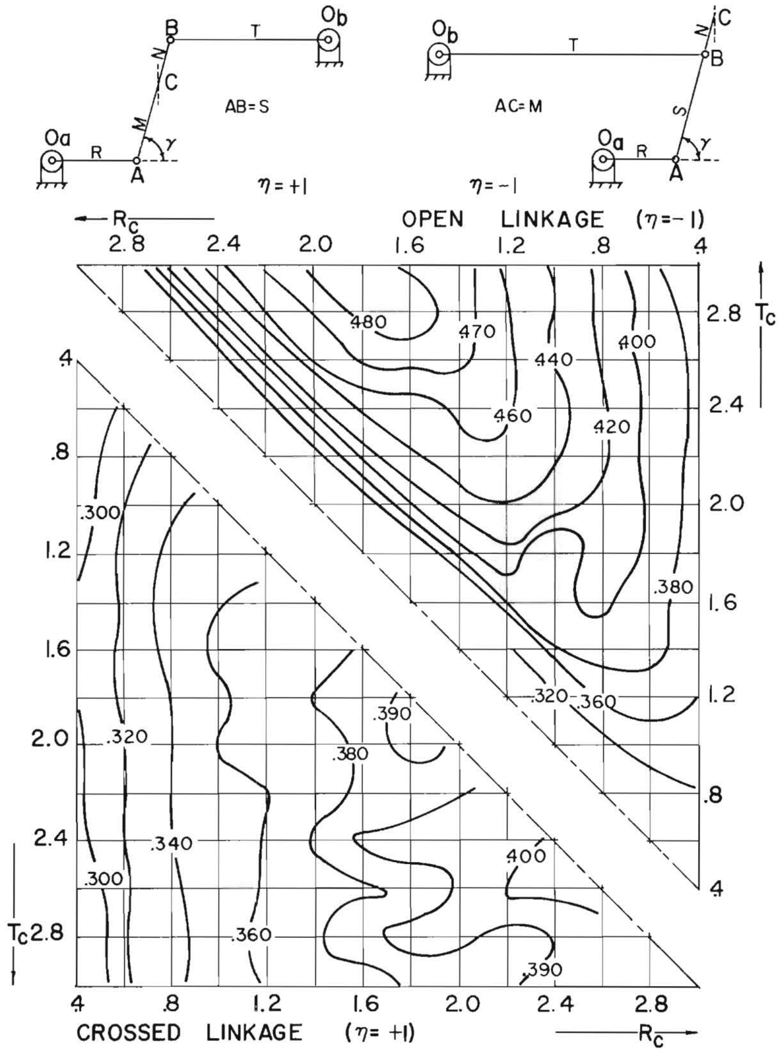
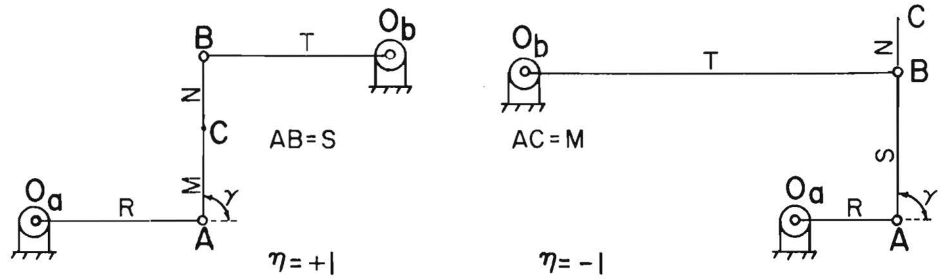


CHART W5, STRAIGHT LINE LENGTH L_c
DEVIATION D_c=0.001 UNIT
GAMMA γ=75°

WATT MECHANISM



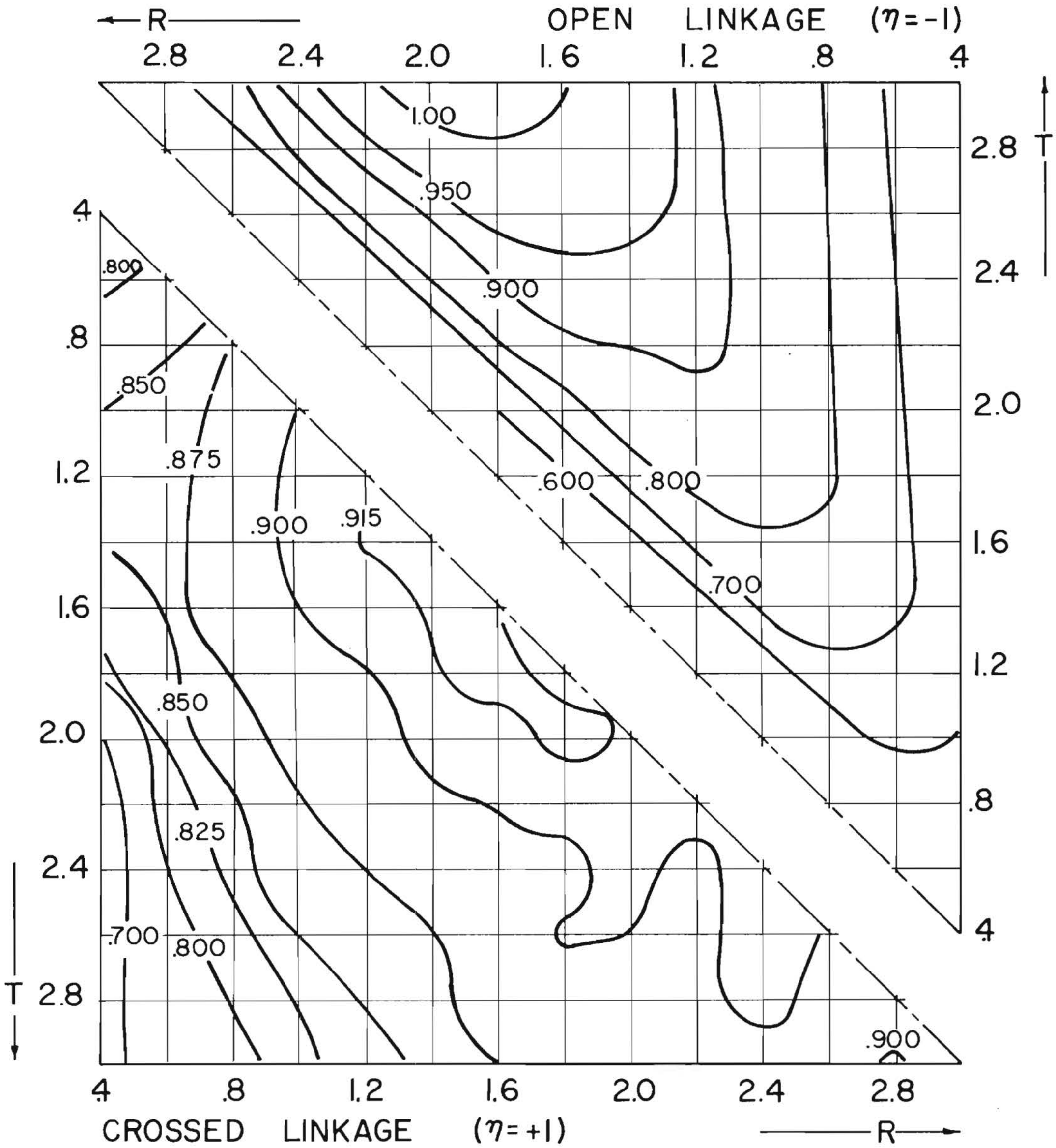
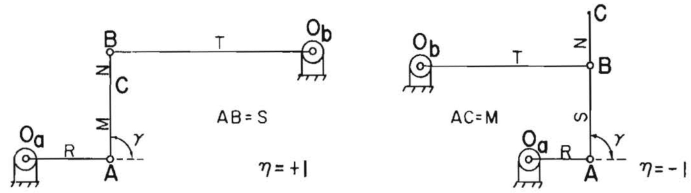


CHART W6, STRAIGHT LINE LENGTH L_c
DEVIATION D =0.01 UNIT
GAMMA η=90°

WATT MECHANISM



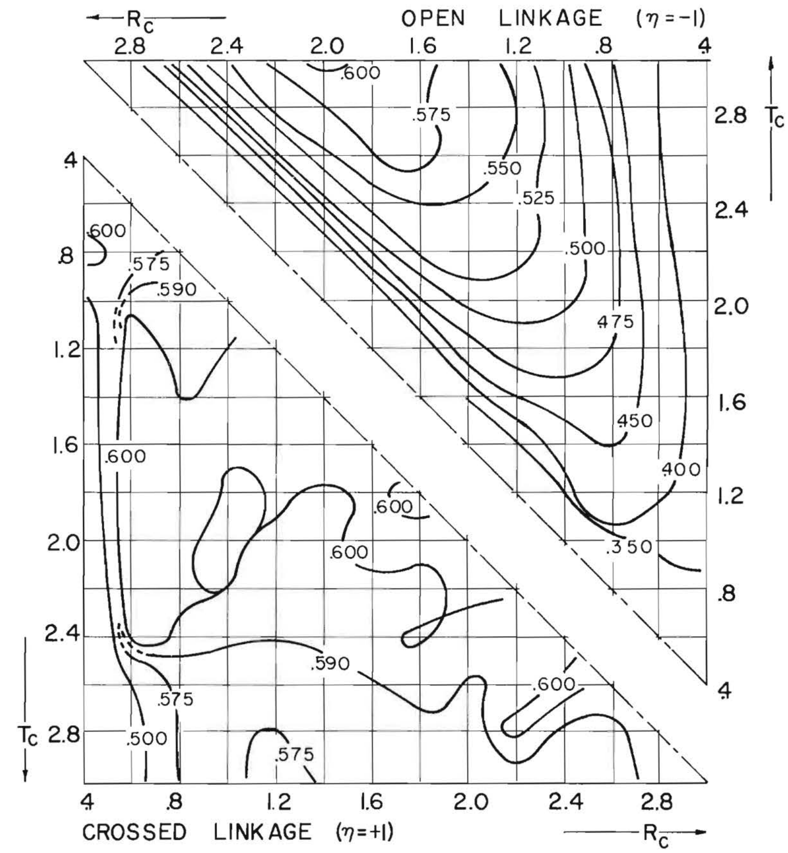


CHART W7, STRAIGHT LINE LENGTH L_C
DEVIATION D_C=0.001 UNIT
GAMMA γ=90°

The Evans Straight-Line Mechanism Case

The Evans mechanism is based on the cardanic circles that produce a special form of cycloidal motion. Points attached to the smaller circle rolling on the inner surface of the twice-the-size fixed circle trace exact straight lines along the diameters of the larger circle. If the straight line path of a point like B Fig. 16 is satisfactorily approximated by a circular arc of sufficiently large radius (O_bB), the resulting four-bar linkage is an Evans mechanism and coupler point D will follow an approximate straight-line path.

Five basic design parameters exist, permitting an astronomically large number of linkages. To reduce the number, limits on these parameters must be imposed. The five parameters and limits used in the investigation are: (a) length of follower crank T denoted by K and limited to ±2; (b) the distance ρ locating the pin joint A on link R and limited to the range +1 to -1; (c) the angle β locating position of fixed point O_a and taken at 0° and 15°; (d) the angle ω which locates the initial O_aA direction and taken at 15, 30, and 45 degrees; and (e) the angle α locating the output coupler point D varying between 0° and 90°. In addition a minimum straight-line length of 0.75 and longest-to-shortest link length of less than 10 are imposed. Based on all of these limitations the most productive ranges were uncovered, computer analyzed and plotted.

Charts E1 to E14 contain the data representing the most fruitful series of mechanisms at deviations of 0.05, 0.01 or 0.001. Applying parameters and other information are well displayed on the charts.

In the case of the Evans mechanism, it becomes necessary to first calculate the coordinates of all pairs, after which linkage dimensions are found by using the general geometrical relation for the distance between two points on a plane

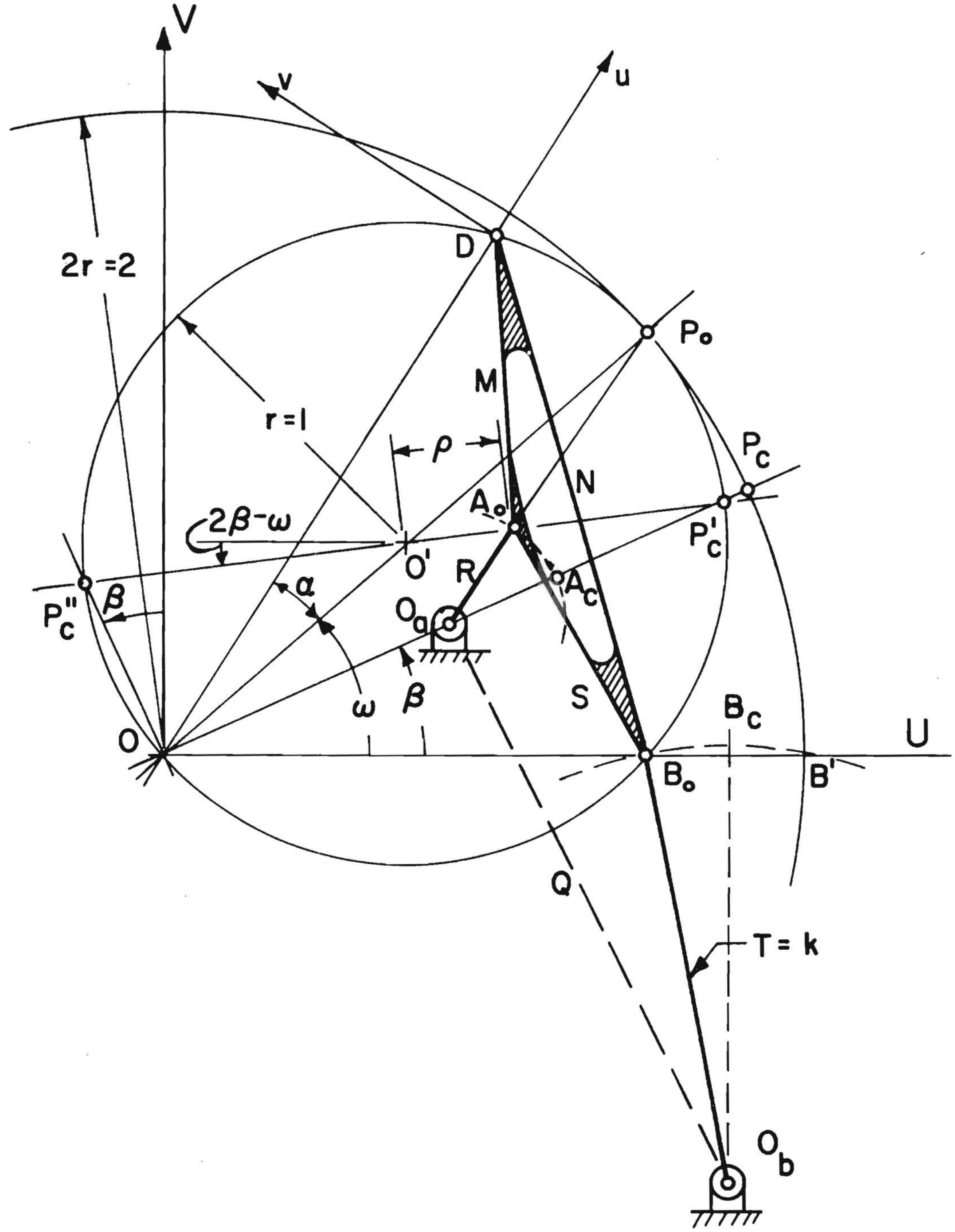


Figure 16. General Form of the Evans Mechanism.

or

$$\text{Length} = \sqrt{(U_2 - U_1)^2 + (V_2 - V_1)^2} \tag{24}$$

where $\underline{U}$ and $\underline{V}$ are measured along the U-V coordinate system shown in Fig. 16.

The coordinates are determined as follows:

The coordinates of $\underline{O}_b$ are

$$U_{O_b} = 1 + \cos \omega \text{ and } V_{O_b} = -k \tag{25}$$

while of pair $\underline{B}$ they are

$$U_B = 2 \cos \omega \text{ and } V_B = 0 \tag{26}$$

The coordinates of $\underline{A}_o$ are

$$U_{A_o} = \cos \omega + \rho \cos (2\beta - \omega) \tag{27}$$

and $V_{A_o} = \sin \omega + \rho \sin (2\beta - \omega)$

The slope of line $\underline{A_oP_o}$ is

$$m = \frac{\sin \omega - \rho \sin (2\beta - \omega)}{\cos \omega - \rho \cos (2\beta - \omega)} \tag{28}$$

such that the equation of line is

$$V = 2 \sin \omega + M(U - 2 \cos \omega) \tag{29}$$

and the equation of $\underline{OP}_c$ is

$$U = V \cot \beta \tag{30}$$

These lines intersect at $\underline{O}_a$ which has the coordinates

$$U_1 = \frac{2 \sin \omega - 2m \cos \omega}{\tan \beta - m}$$

and (31)

$$V_1 = \frac{2 \sin \omega - 2m \cos \omega}{1 - m \cot \beta}$$

Point D is thus located at

$$U_D = 2 \cos (\alpha + \omega) \cos \alpha$$

(32)

$$V_D = 2 \sin (\alpha + \omega) \cos \alpha$$

Having located the coordinates of all pertinent points, the link dimensions can be readily found by Eq. 24.

Example

An example will illustrate the use of the charts. Select any point on any chart, say,

$$K = -2.0$$

$$\omega = 30°$$

$$\beta = 15°$$

$$\rho = 0.1$$

$$\alpha = 60°$$

$$L_c = 1.98$$

$$D_c = 0.05$$

Calculating the coordinates in the U, V system of the pin joints - using Eq. (28)

$$m = \frac{\sin 30 - (0.1) \sin (2 \times 15 - 30)}{\cos 39 - (0.1) \cos (2 \times 15 - 30)} = 0.653$$

and Eqs. (31)

$$V_1 = \frac{2 \sin 30 - 2(0.653) \cos 30}{1 - (0.653) \cot 15} = 0.091$$

Then by Eq. (30)

$$U_1 = V_1 \cot\beta = (0.091) \cot 15 = 0.33$$

By Eqs. (25)

$$V_{O_b} = -k = +2.0$$

$$U_{O_b} = 1 + \cos\omega = 1 + \cos 30 = 1.866$$

The coordinates of A_o are by Eqs. (27)

$$V_{A_o} = \sin 30 + (0.1) \sin [2(15) - 30] = 0.500$$

and

$$U_{A_o} = \cos 30 + (0.1) \cos [2(15 - 30)] = 0.966$$

Those of point B are by Eqs. (26)

$$U_B = 2 \cos\omega = 2 \cos 30 = 1.732$$

$$V_B = 0$$

The coordinates of coupler point $\underline{D}$ are by Eqs. (32)

$$U_D = 2 \cos (60) \cos (60 + 30) = 0.0$$

$$V_D = 2 \cos (60) \sin (60 + 30) = 0.500$$

Then calculating link dimensions by Eq. (24)

$$Q_c = \sqrt{(U_2 - U_1)^2 + (V_2 - V_1)^2}$$

$$= \sqrt{(1.866 - 0.33)^2 + (2.00 - 0.091)^2} = 2.44$$

$$R_c = 0.74$$

$$S_c = 0.91$$

$$T_c = 2.00$$

$$M_c = 1.09$$

$$N_c = 2.00$$

Therefore

$$UN = \frac{Q + T + S + T + \frac{M + N}{2}}{5} = 1.53$$

and

$$Q = Q_c/UN = \frac{2.44}{1.53} = 1.60$$

$$R = R_c/UN = 0.49$$

$$S = S_c/UN = 0.60$$

$$T = T_c/UN = 1.31$$

$$M = M_c/UN = 0.71$$

$$N = N_c/UN = 0.31$$

These dimensions define the mechanism which will produce the specified output, a length of straight line $L_c = 1.98$ within a deviation of $D_c = 0.05$ shown in Fig. 17.

PARAMETERS

$k = -2.0 \quad \omega = 30° \quad \beta = 15° \quad \rho = 0.1 \quad \alpha = 60°$

DIMENSIONS

Q	=	1.5964	S	=	0.5974
R	=	0.4888	M	=	0.7104
T	=	1.3091	N	=	1.3062

RESULTS

D	0.001	0.01	0.05
L	0.109	1.364	1.975

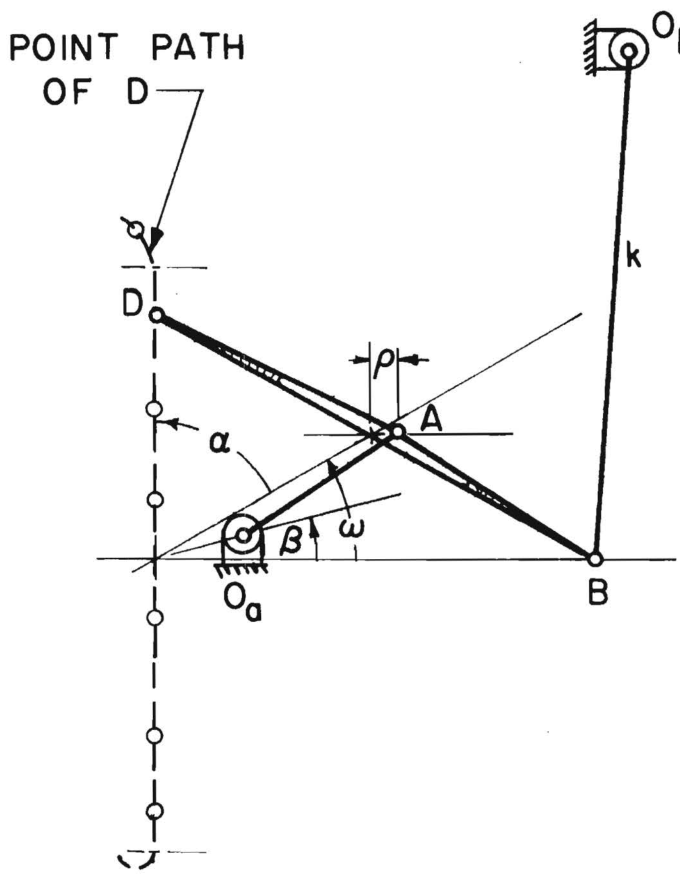
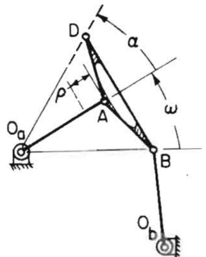


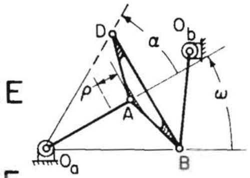
Figure 17. Example, Evans Mechanism.

EVANS MECHANISM



1.3 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

5.0 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME RATIO OF LONGEST TO SHORTEST LINK



k = 2.0 ω = 30 β = 0

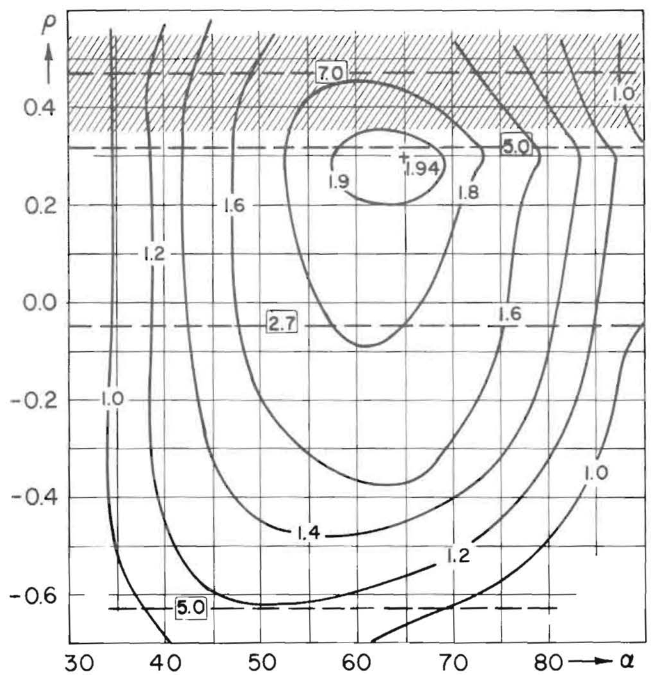


CHART E 1

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISMS, OTHER AREAS REPRESENT DOUBLE LEVER MECHANISMS

k = −2.0 ω = 30 β = 0

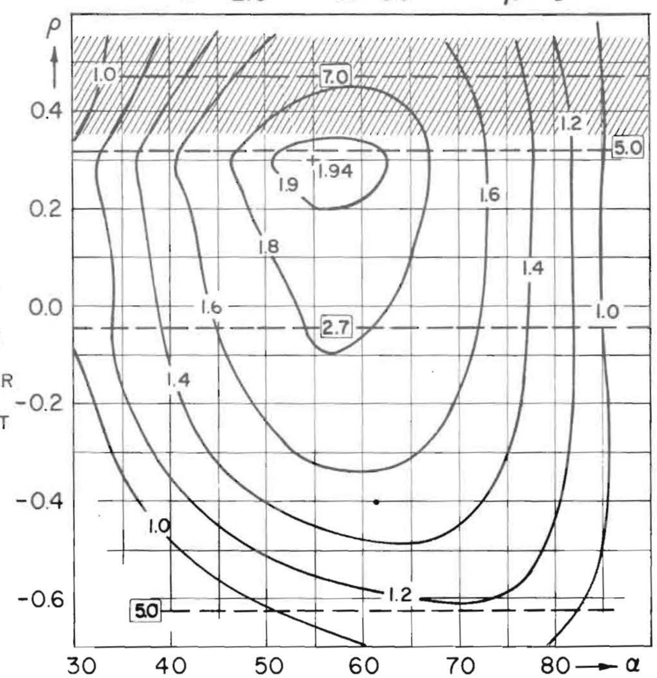
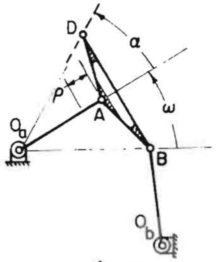
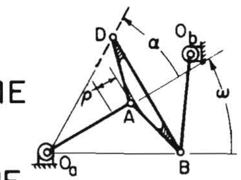


CHART E 2

STRAIGHT LINE LENGTH L_c FOR DEVIATION D_c=0.05 UNIT AND RATIO OF LONGEST TO SHORTEST LINK



EVANS MECHANISM



1.2 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

5.0 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME RATIO OF LONGEST TO SHORTEST LINK

k = 2.0 ω = 30 β = 0

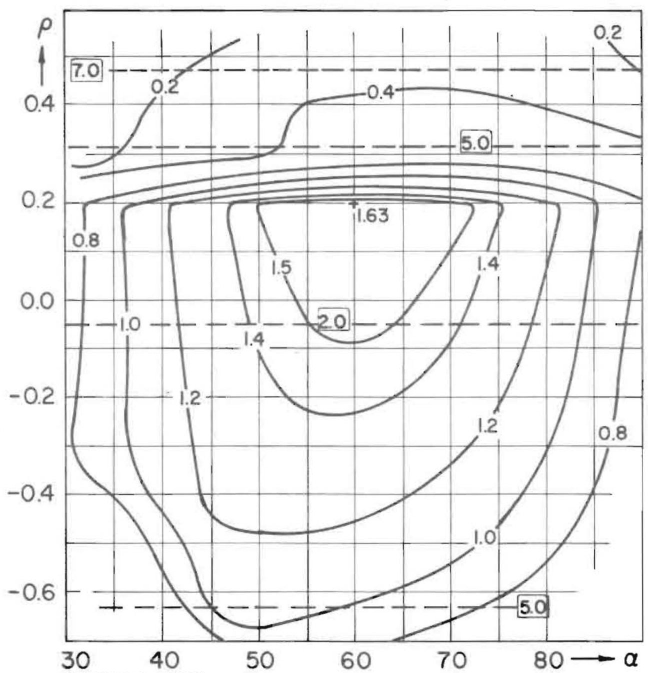


CHART E 3

k=−2.0 ω = 30 β = 0

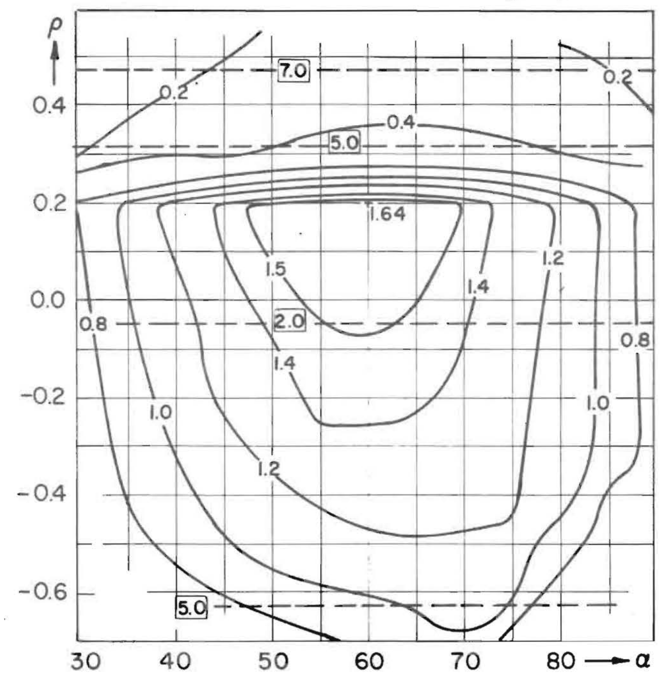
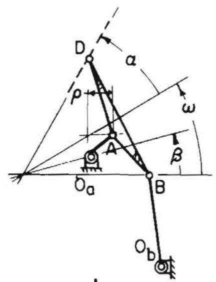


CHART E 4

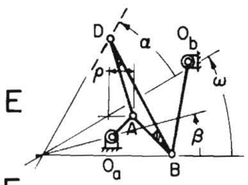
STRAIGHT LINE LENGTH L_c FOR DEVIATION D_c=0.01 UNIT AND RATIO OF LONGEST TO SHORTEST LINK

EVANS MECHANISM



1.3 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

5.0 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME RATIO OF LONGEST TO SHORTEST LINK



k = 2.0 ω = 30 β = 15

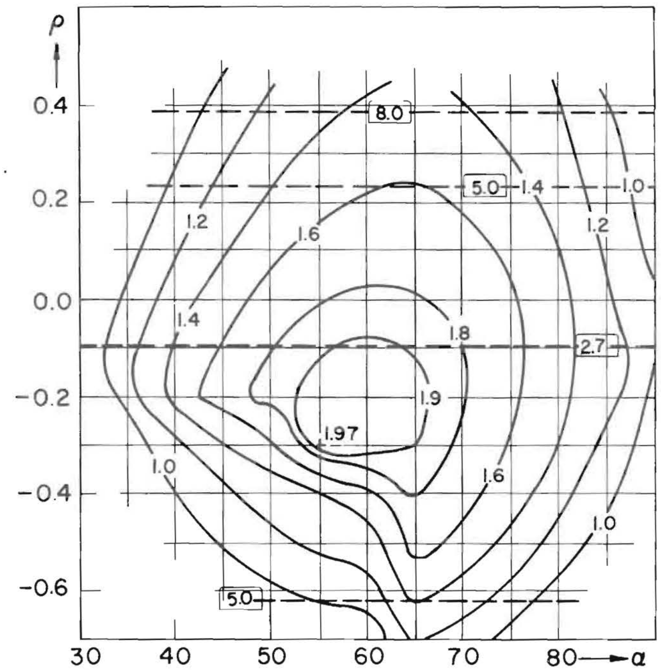


CHART E5

CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISMS, OTHER AREAS REPRESENT DOUBLE LEVER MECHANISMS

k = -2.0 ω = 30 β = 15

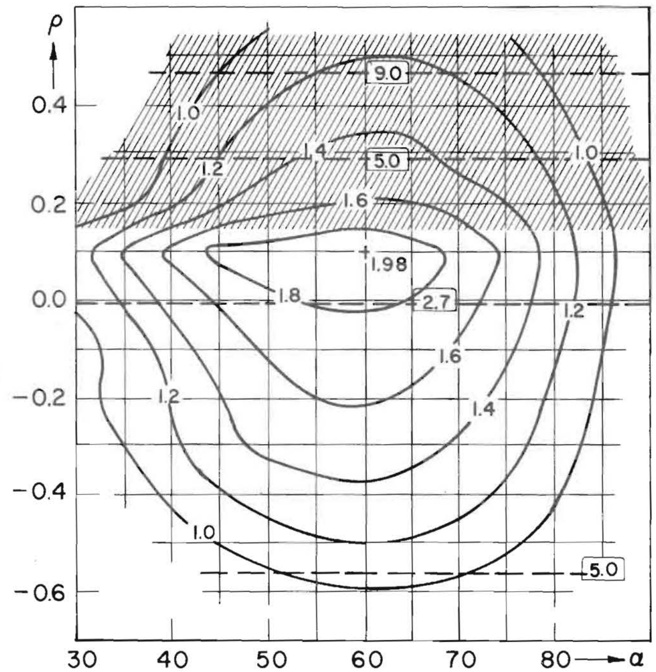
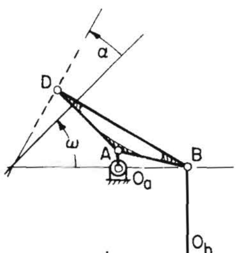


CHART E6

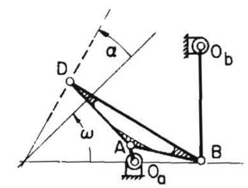
STRAIGHT LINE LENGTH L_c FOR DEVIATION D_c=0.05 UNIT AND RATIO OF LONGEST TO SHORTEST LINK

EVANS MECHANISM



CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINKS VARIES FROM 2.59 TO 5.03



$k = 2.0$ $\omega = 45$ $\beta = 0$

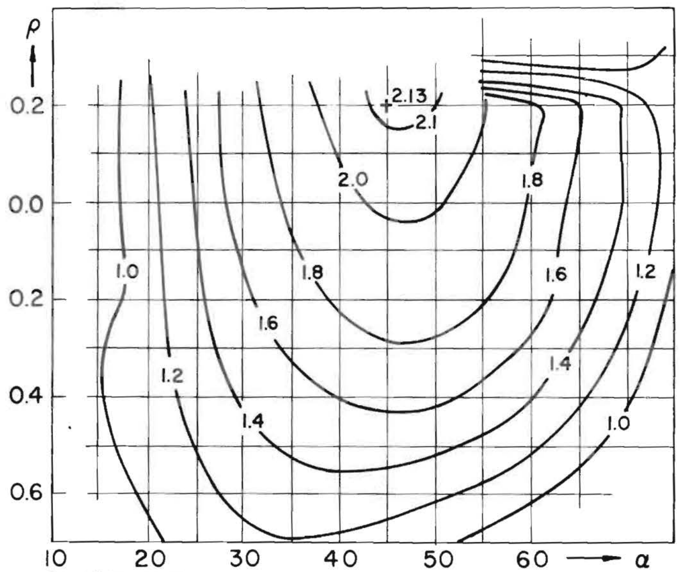


CHART E 7

$k = -2.0$ $\omega = 45$ $\beta = 0$

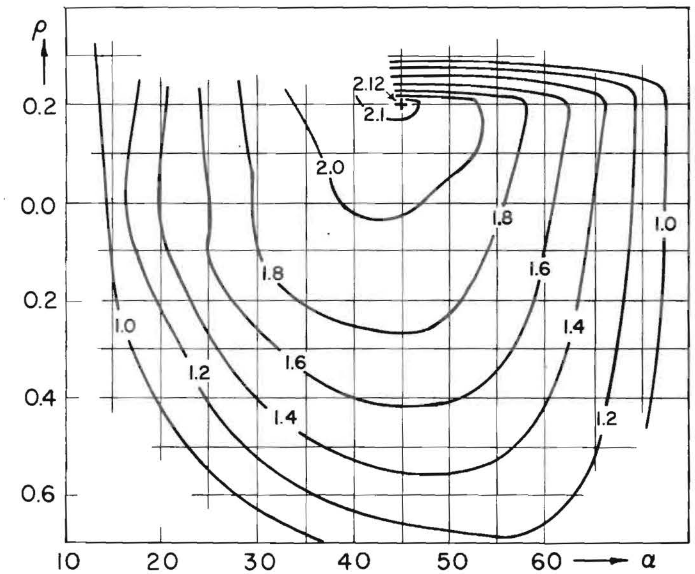


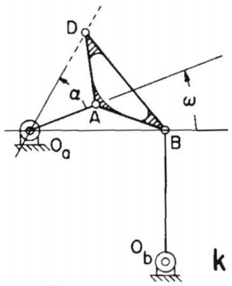
CHART E 8

STRAIGHT LINE LENGTH L_c FOR DEVIATION $D_c = 0.05$

EVANS MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINKS VARIES FROM 2.50 TO 2.83



k = 2.0 ρ = 0.0

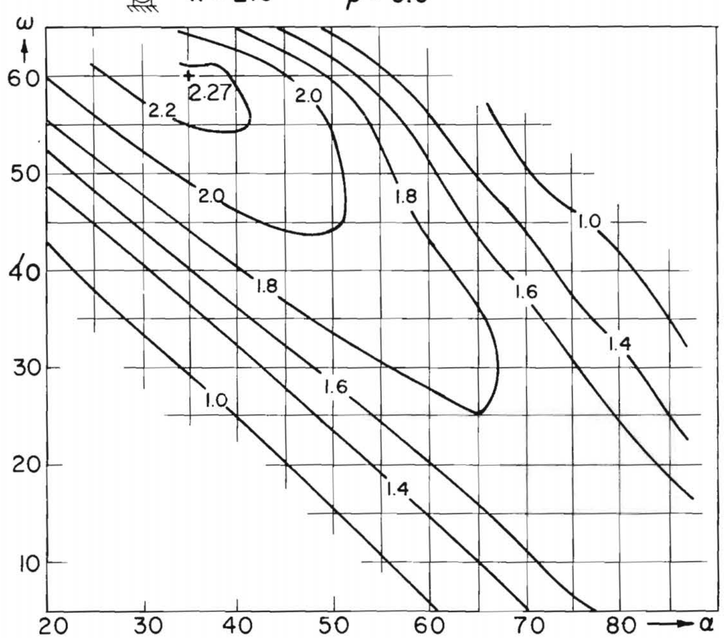
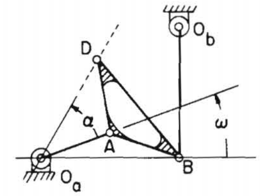


CHART E 9



k = −2.0 ρ = 0.0

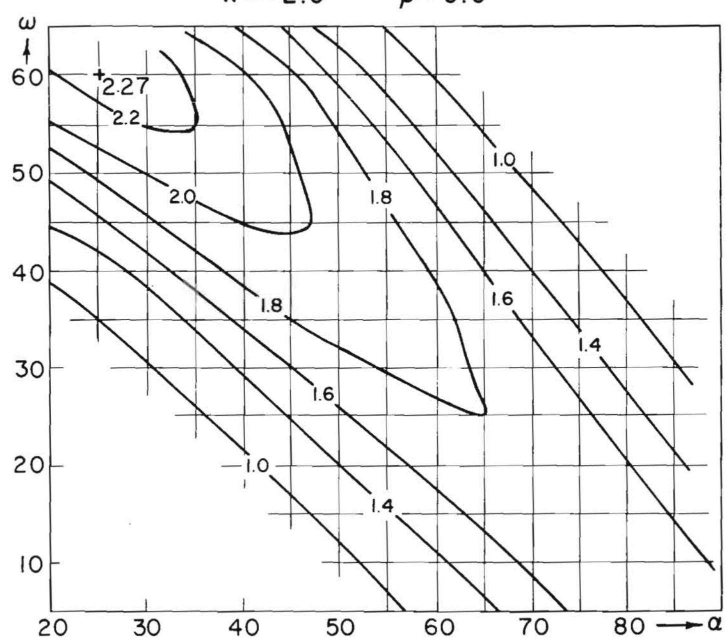


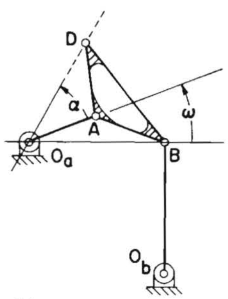
CHART E 10

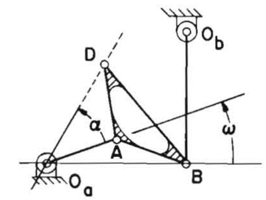
STRAIGHT LINE LENGTH L_c FOR DEVIATION D_c = 0.05

EVANS MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINKS VARIES FROM 2.50 TO 2.83





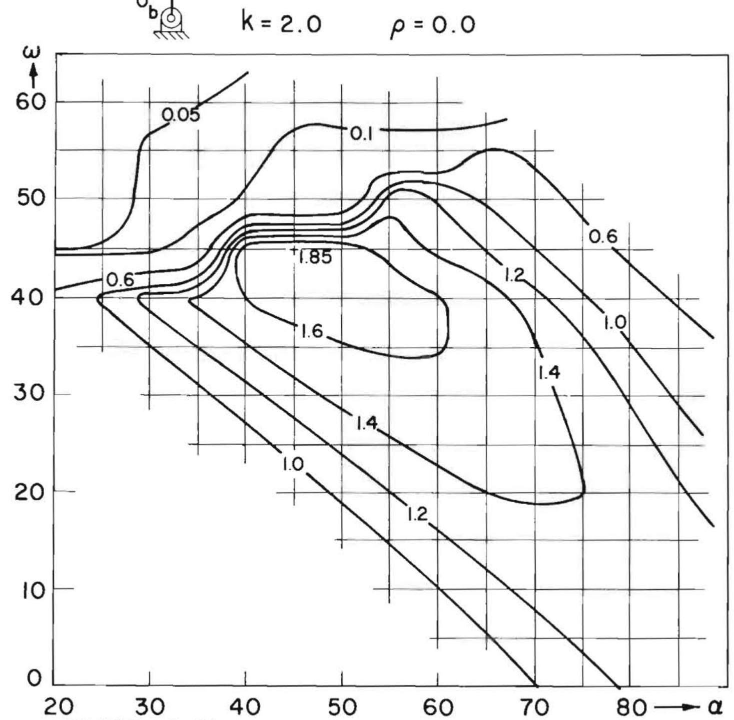


CHART E 11

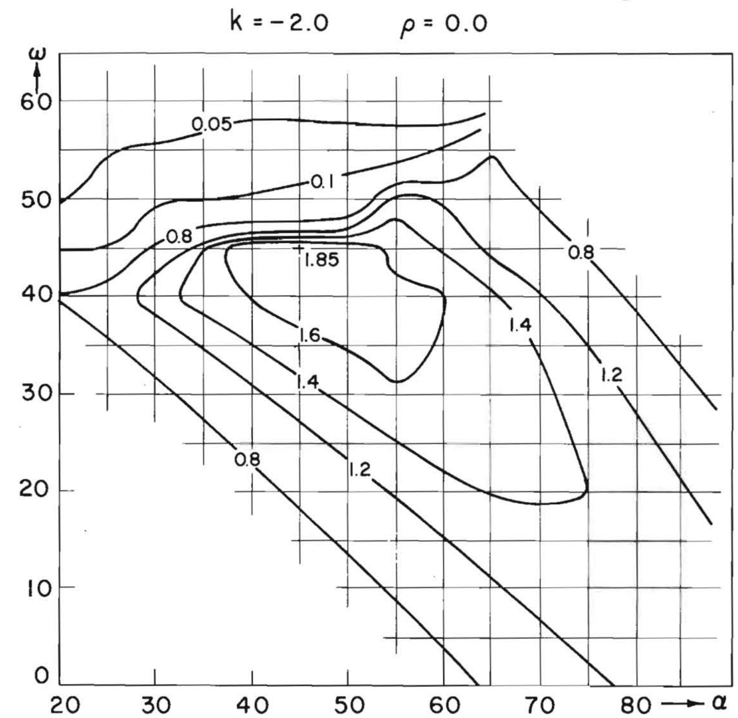
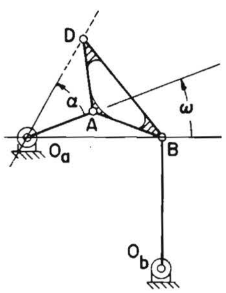


CHART E 12

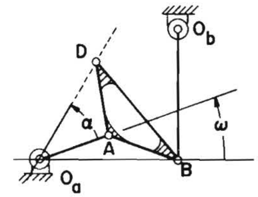
STRAIGHT LINE LENGTH L_c FOR DEVIATION D_c = 0.01

EVANS MECHANISM



CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINKS VARIES FROM 2.50 TO 2.83



$k = 2.0 \quad \rho = 0.0$

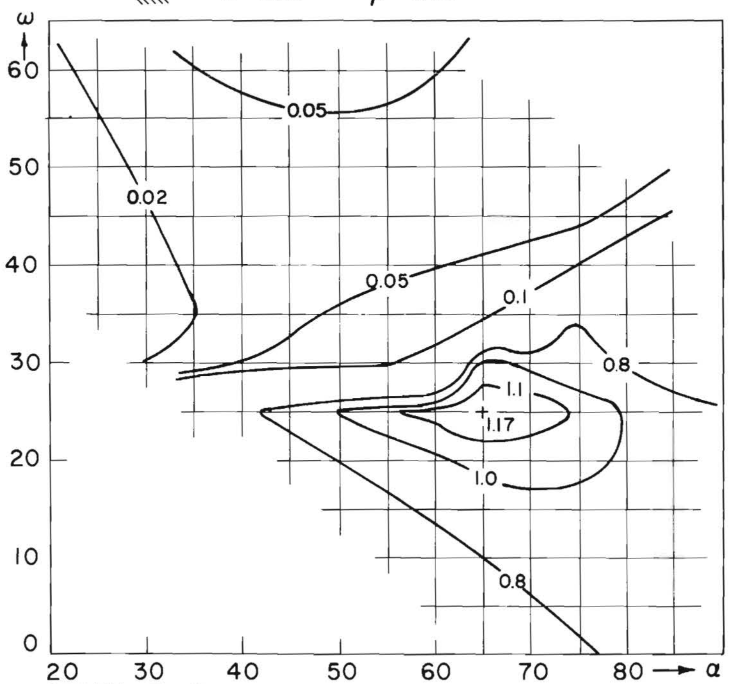


CHART E 13

$k = -2.0 \quad \rho = 0.0$

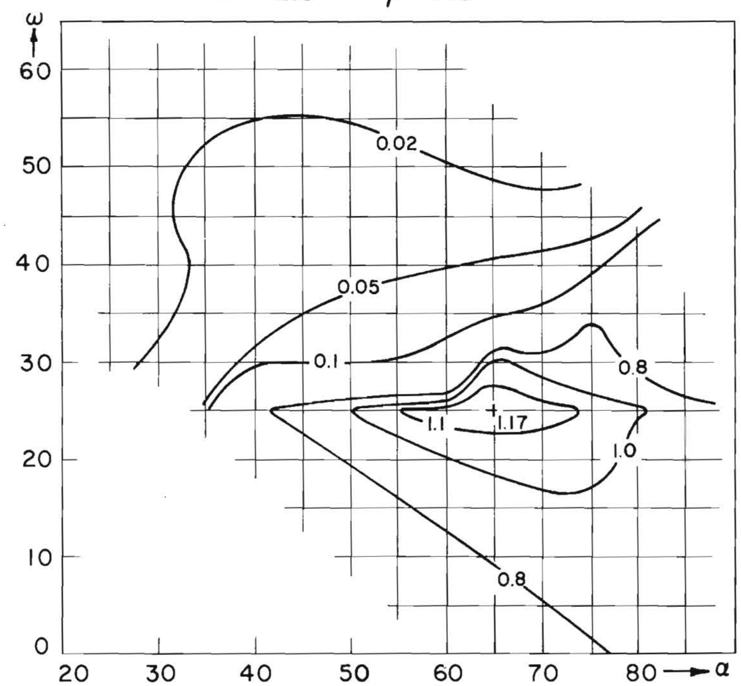


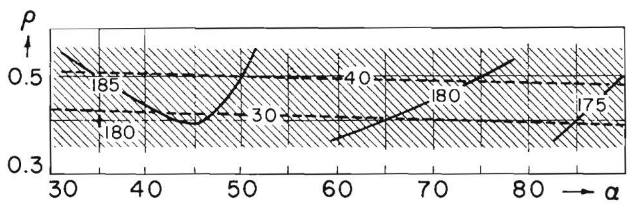
CHART E 14

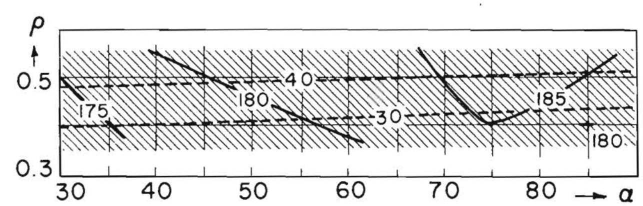
STRAIGHT LINE LENGTH L_c FOR DEVIATION $D_c = 0.001$

EVANS MECHANISM

—185— CONTOUR LINES REPRESENT CRANK AND LEVER MECHANISMS HAVING THE SAME CRANK ROTATION ANGLE REQUIRED TO GENERATE THE APPROXIMATE STRAIGHT LINE.

- - -30- - - CONTOUR LINES REPRESENT CRANK AND LEVER MECHANISMS HAVING THE SAME MINIMUM VALUE FOR THE TRANSMISSION ANGLE AT THE FOLLOWER PIN JOINT WHICH OCCURS DURING THE GENERATION OF THE APPROXIMATE STRAIGHT LINE.





CROSSHATCHED AREAS REPRESENT CRANK AND LEVER MECHANISMS, OTHER AREAS REPRESENT DOUBLE LEVER MECHANISMS

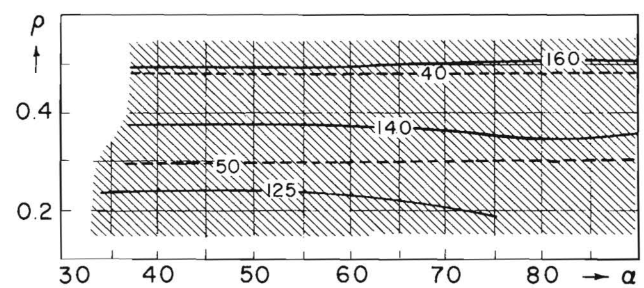


CHART E 15

CRANK ROTATION ANGLES AND FOLLOWER PIN JOINT TRANSMISSION ANGLES FOR DEVIATION $D_c = 0.05$

The Symmetric Mechanisms Case

The symmetrical linkages, such as shown in Fig. 18, are geometrically quite simple. All such linkages are specified by the following three parameters shown in the figure.

a: This parameter orients the fixed pivots about the center line of the linkage. If negative the cranks are crossed.

b: This parameter identifies the height of the coupler triangle.

c: This parameter specifies the distance from the coupler line AB to the line of fixed-link centers with the mechanism in its central position.

Charts S1 to S12 contain the straight-line lengths at deviations of 0.05 and 0.001 and specified a values extending between a = +5 and a = -5.

Link dimensions which provide all the formulation needed are based on the coupler link AB taken as S = 2.00.

$$M = N = [1 + c^2]^{\frac{1}{2}}$$

$$\epsilon = \tan^{-1}(-c) \tag{33}$$

$$Q = 2a$$

and

$$R = T = [d^2 + (1-a)^2]^{\frac{1}{2}}$$

Example

Choose any point on any chart such as

$$a = -3.0$$

$$c = 1.0$$

$$d = 2.5$$

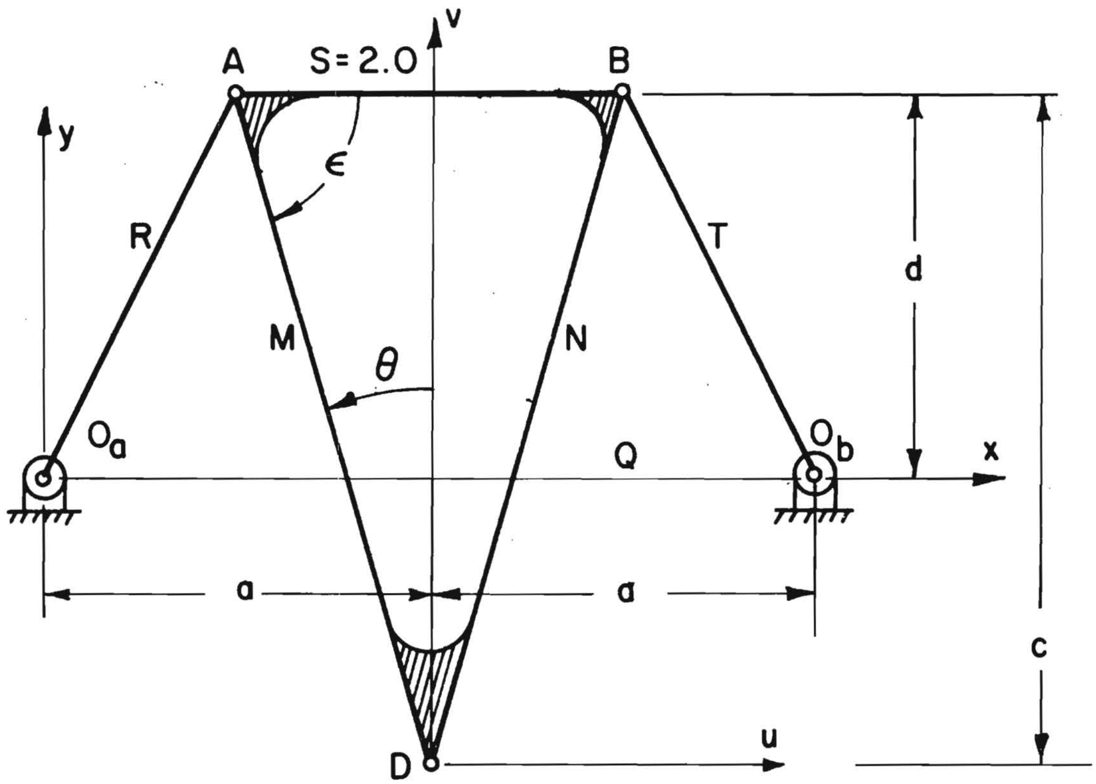


Figure 18. Symmetrical (Roberts) Mechanism.

$$L_c = 1.13$$

$$D_c = 0.05$$

Since a = -3.0, this is a mechanism with crossed cranks. Calculating the dimensions from equation (33)

$$S_c = 2.0$$

$$Q_c = |2_a| = |2 \times -3.0| = 6.0$$

$$R_c = T_c = [d^2 + (1 - a)^2]^{\frac{1}{2}}$$

$$= [(2.5)^2 + (1 + 3.0)^2]^{\frac{1}{2}} = 4.71$$

$$M_c = N_c = [1 + c^2]^{\frac{1}{2}}$$

$$= [1 + 1^2]^{\frac{1}{2}} = 1.42$$

and using Eq. (3)

$$UN_c = \frac{2 + 6 + 4.71 + 4.71 + (1.42 + 1.42)/2}{5} = 3.77$$

Hence:

$$Q = Q_c/UN = 6.0/3.77 = 1.59$$

$$R = R_c/UN = 1.25$$

$$S = S_c/UN = 0.53$$

$$T = T_c/UN = 1.25$$

$$M = N = M_c/UN = 0.38$$

This is a symmetrical mechanism which will produce the specified straight lines at the specified deviation, as shown in Fig. 19.

PARAMETERS

$a = -3.0 \quad c = 1.0 \quad d = 2.5$

DIMENSIONS

Q	=	1.5917	S	=	0.5306
R	=	1.2513	M	=	0.3752
T	=	1.2513	N	=	0.3752

RESULTS

D	0.001	0.01	0.05
L	0.768	0.893	1.130

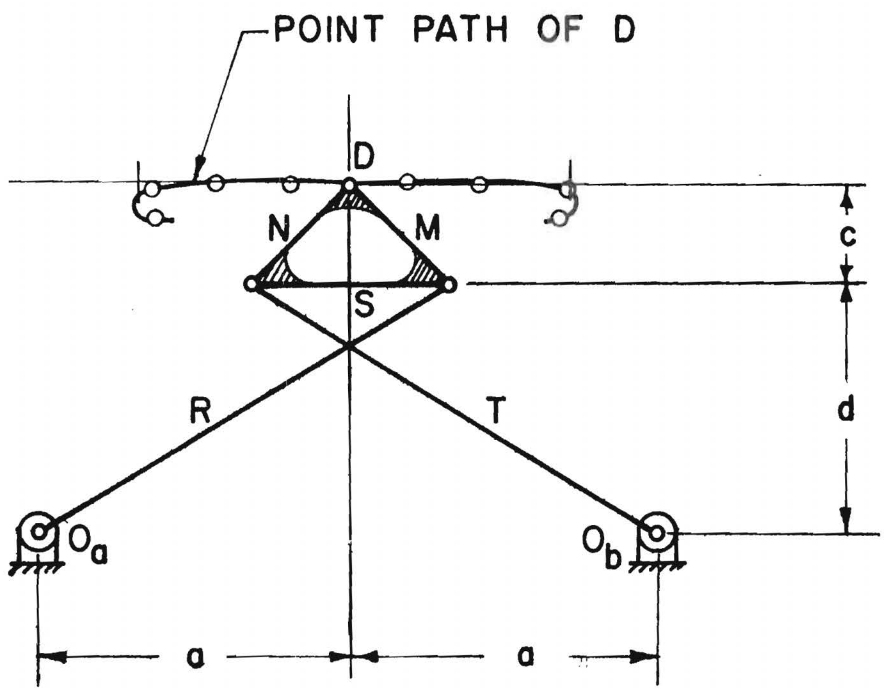
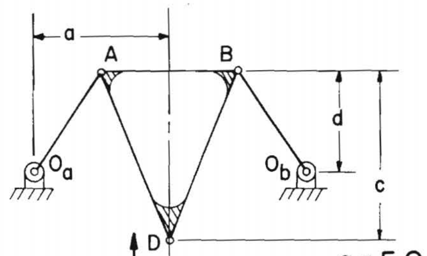


Figure 19. Example, Symmetrical (Chebishev) Mechanism.



SYMMETRICAL MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINK VARIES FROM 3.0 TO 3.6

a = 5.0

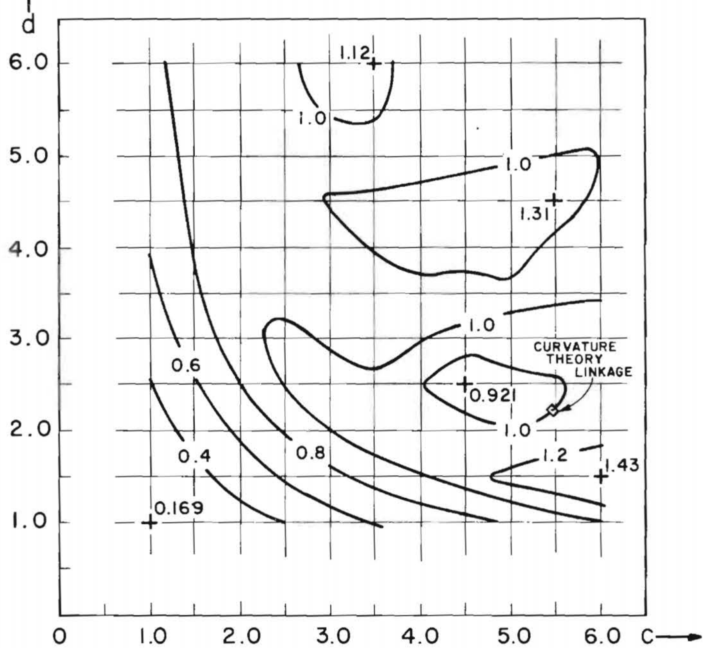


CHART S 1 DEVIATION D_c = 0.05 UNIT

a = 5.0

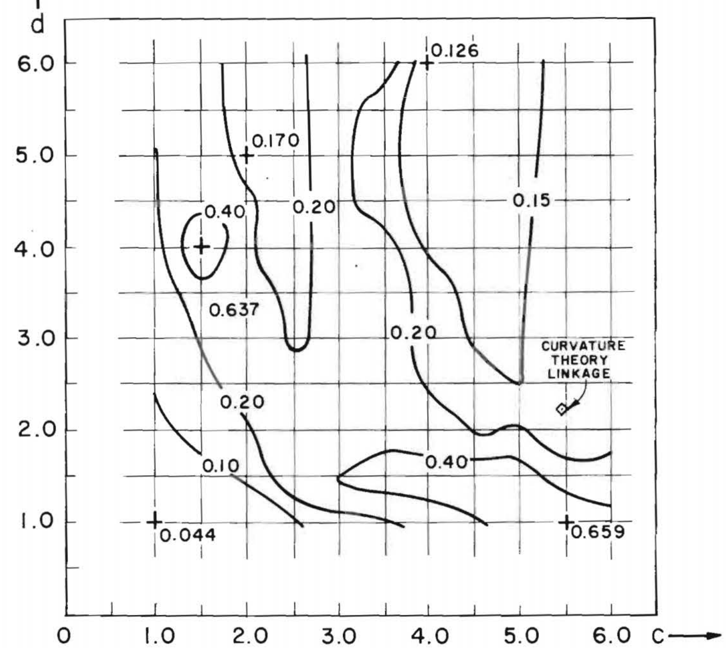
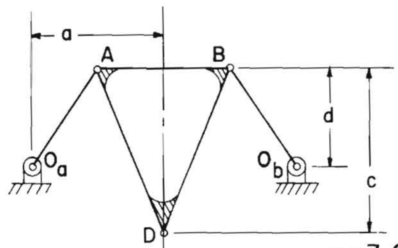


CHART S 2 DEVIATION D_c = 0.001 UNIT

STRAIGHT LINE LENGTH L_c AND RATIO OF LONGEST TO SHORTEST LINK



SYMMETRICAL MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINK VARIES FROM 3.0 TO 3.6

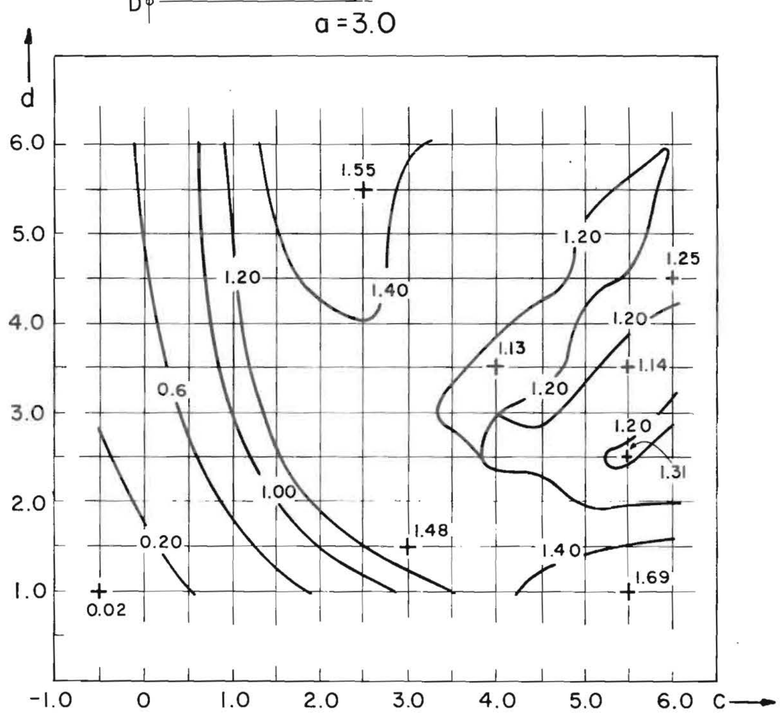


CHART S 3 DEVIATION D_c = 0.05 UNIT

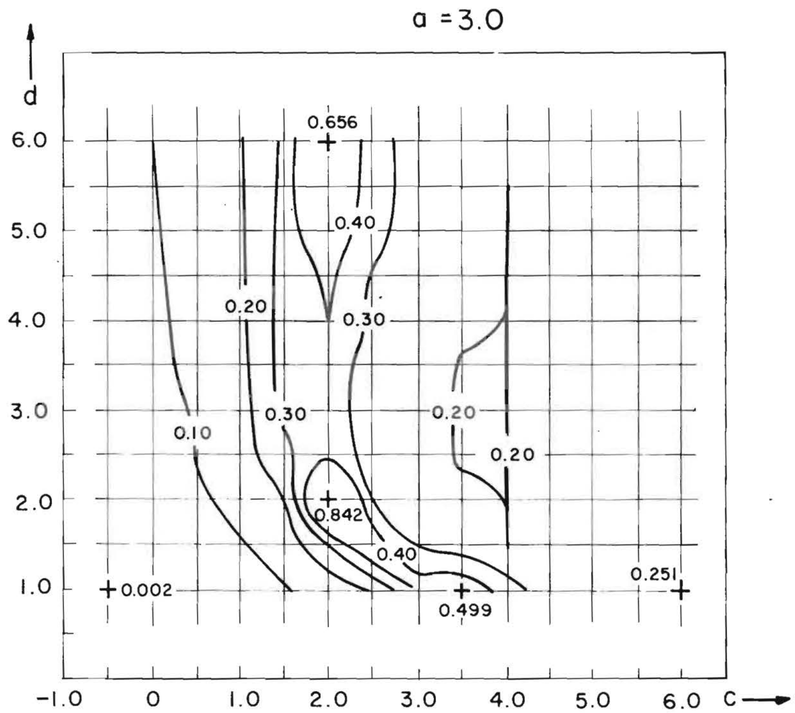
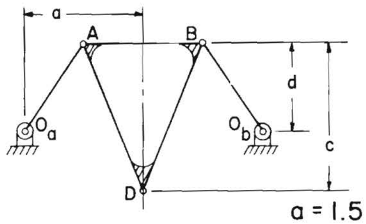


CHART S 4 DEVIATION D_c = 0.001 UNIT

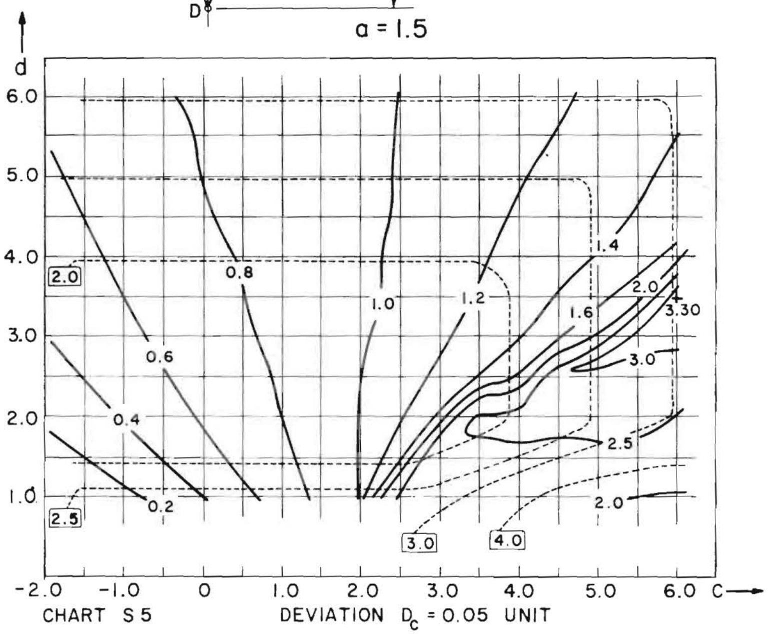
STRAIGHT LINE LENGTH L_c AND RATIO OF LONGEST TO SHORTEST LINK

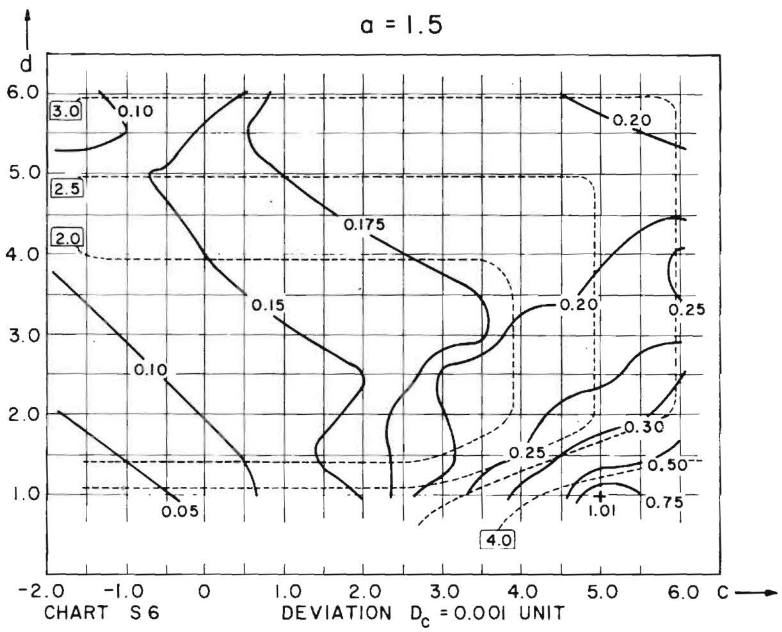
SYMMETRICAL MECHANISM



1.2 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

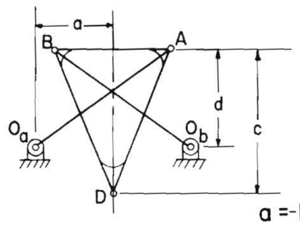
2.0 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME RATIO OF LONGEST TO SHORTEST LINK





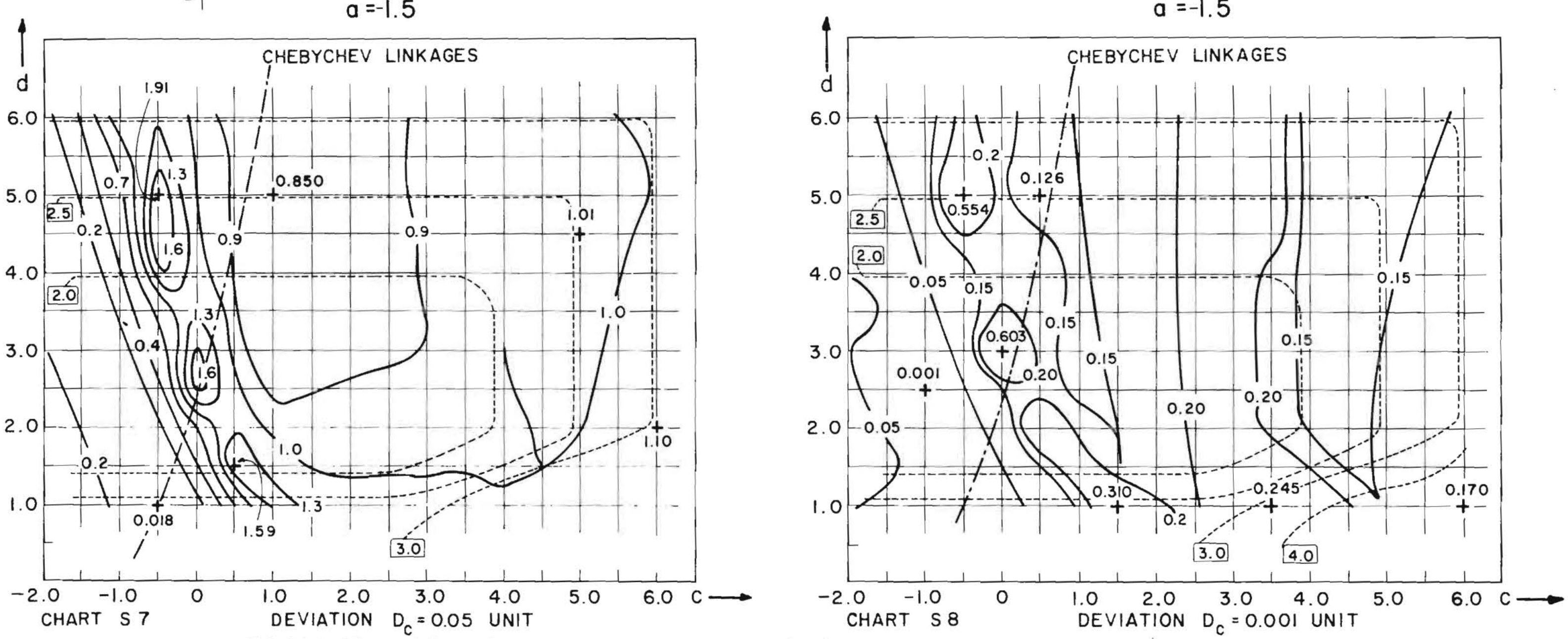
STRAIGHT LINE LENGTH L_c AND RATIO OF LONGEST TO SHORTEST LINK

SYMMETRICAL MECHANISM

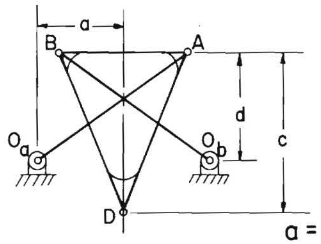


1.2 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

2.0 CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME RATIO OF LONGEST TO SHORTEST LINK



STRAIGHT LINE LENGTH L_c AND RATIO OF LONGEST TO SHORTEST LINK



SYMMETRICAL MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINK VARIES FROM 3.0 TO 3.6

a = -3.0

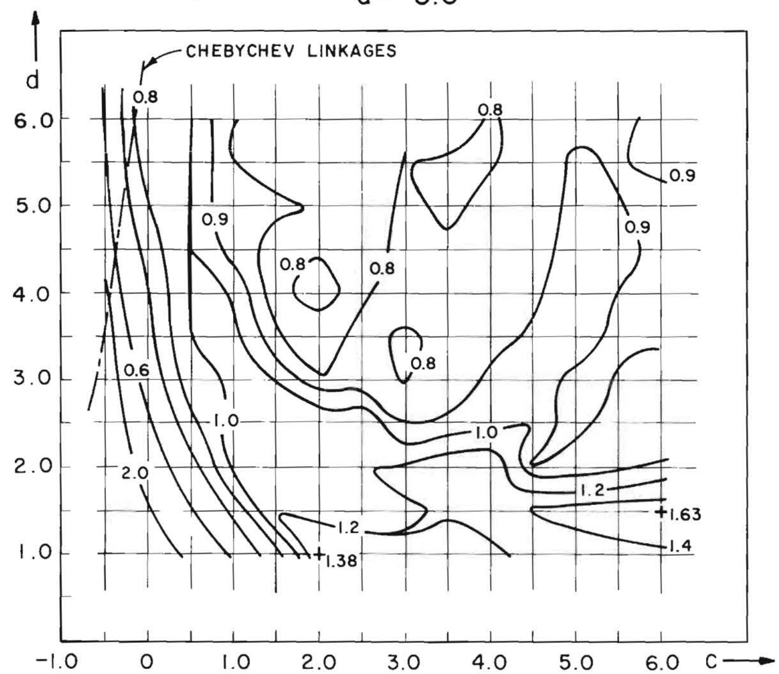


CHART S 9 DEVIATION D_c = 0.05 UNIT

a = -3.0

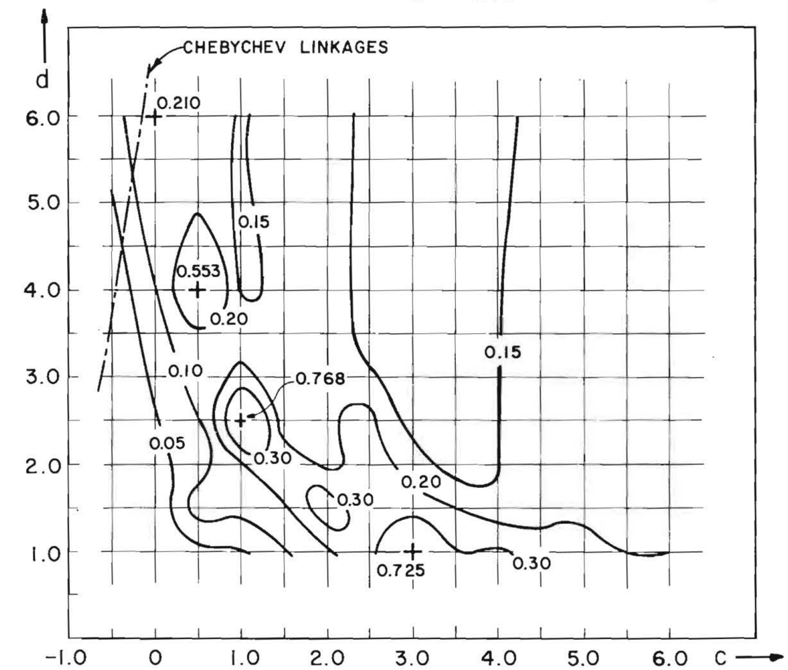
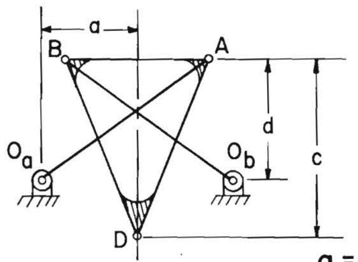


CHART S 10 DEVIATION D_c = 0.001 UNIT

STRAIGHT LINE LENGTH L_c AND RATIO OF LONGEST TO SHORTEST LINK



SYMMETRICAL MECHANISM

CONTOUR LINES REPRESENT LINKAGES HAVING THE SAME LENGTH OF APPROXIMATE STRAIGHT LINE OUTPUT

RATIO OF LONGEST TO SHORTEST LINK VARIES FROM 3.0 TO 3.6

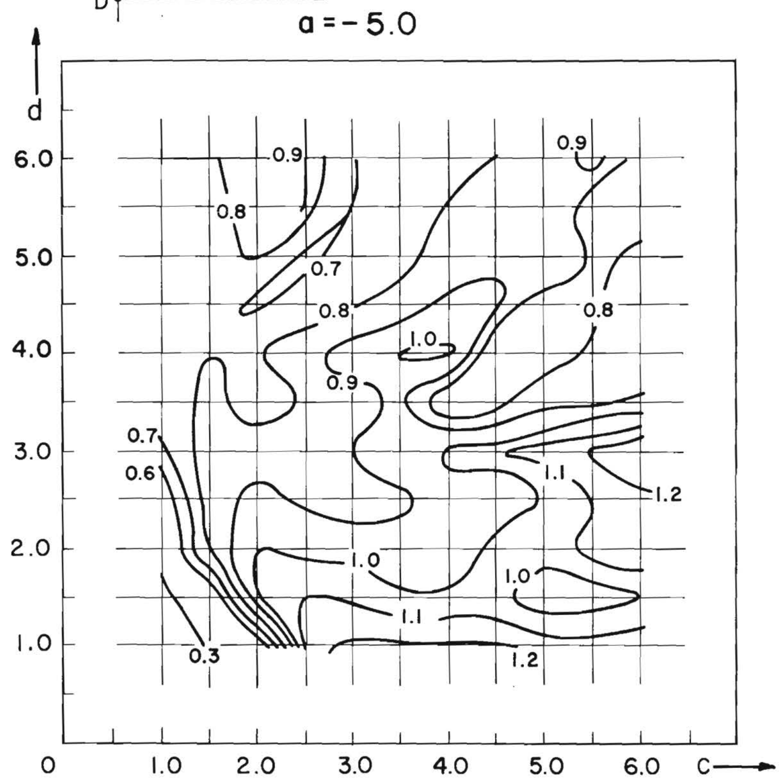


CHART S11 DEVIATION $D_c = 0.05$

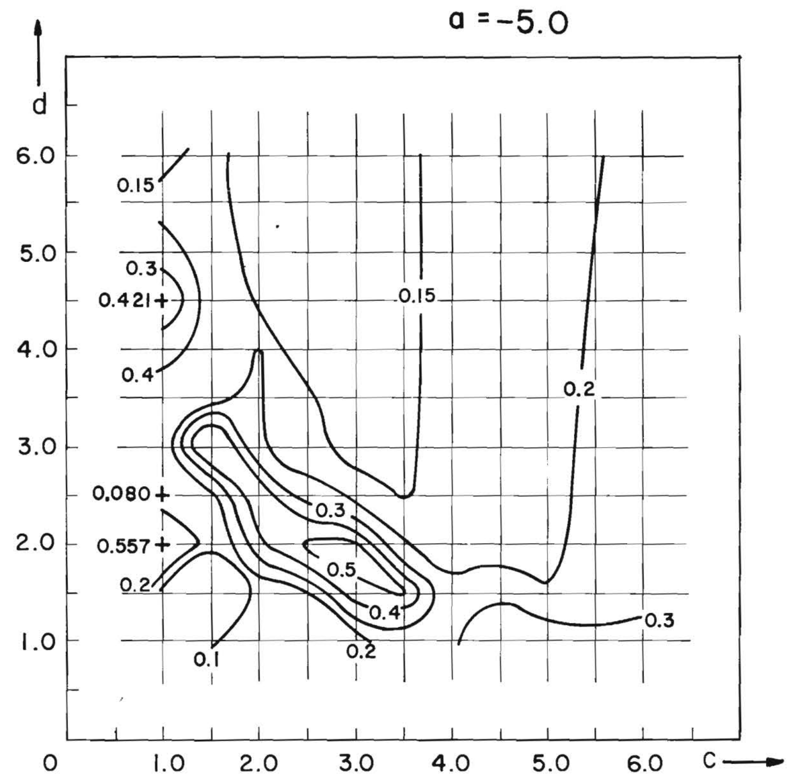


CHART S 12 DEVIATION $D_c = 0.001$

STRAIGHT LINE LENGTH L_c AND RATIO OF LONGEST TO SHORTEST LINK

The Ball-Burmester Point - Finite Intersection Case

The data obtained for this case are rather limited. The complexity of the situation lies in the fact that four parameters and two sets of charts are involved in the selection. Nevertheless, when these charts are used together with the general Ball-Burmester set, BB1 to BB36, a great variety of approximate straight-line linkages of this type become available.

Charts FI1 to FI7 present the alternate A,B mechanisms for seven different values of distance between the Ball-Burmester point and the intersection point. Charts FI8A to FI13 deal with the B,C alternate for six different point-to-intersection distance values.

Equation (3) through (7) of the General Ball-Burmester point case apply here as well. An example will illustrate computations and use of charts. FI together with the BB charts.

Example

Suppose it is desired that the intersection point be 0.6 unit distant from the Ball-Burmester point and at an accuracy of 0.01 unit.

Several combinations of parameters are available on charts FI1 and FI12 at the desired value of S_{12}. Note, however, that the selection is limited to values of α_d for which Ball-Burmester (BB) charts are available. Thus choose, say

$$\alpha_d = 55°, \ \alpha_a = 22°, \text{ and } \alpha_b = 10°$$

on chart FI12, alternate B,C.

Then on chart BB15 a linkage of above parameters having an approximate straight-line output of 1.42 units is found. The crank rotation angle of 205°

(chart BB16) and minimum transmission angle of 24° (chart BB17) appear satisfactory as well.

Calculations using Eqs. (4) to (7) and proceeding as in the Ball-Burmester case yield

$$V = 0.4040 + 0.1763 = 0.5803$$

$$W = (0.4040)\ (0.1763) = 0.0712$$

$$PA = \frac{[(3 \times 0.0712 + 1)\ 1.4281 + 0.5803 \times 0.0712]\ 0.3746}{(0.0712 + 1)\ 0.4040 + 0.0712\ (2 \times 1.4281 + 0.5803)} = 0.9811$$

$$PB = \frac{[1.7744]\ 0.1736}{(1.0712)\ 0.1763 + 0.2447} = 0.7105$$

$$\alpha_c = \tan^{-1}\left\{- \frac{2 \times 1.4281 + 0.5803}{0.0712 + 1}\right\} = -72.7° \text{ or } 107.3°$$

and using the α_c between 0° and 180°

$$PC = \frac{[1.7744]\ 0.9548}{(0.0712 - 1)\ (2 \times 1.4281 + 0.5803)} = -0.5307$$

Thus O_b and O_c are at

$$PO_b = - \frac{0.7105 \times 0.1736}{0.7105 - 0.1736} = -0.2297$$

$$PO_c = - \frac{(-0.5307)\ 0.9548}{-0.5307 - 0.9548} = -0.3411$$

$$PD = \sin 55 = 0.8192$$

The linkage dimensions are

$$O_bB = \frac{(0.7105)^2}{0.7105 - 0.1736} = 0.9402$$

$$O_cC = \frac{(-0.5307)^2}{-0.5307 - 0.9548} = -0.1896$$

$$BC = \{(-0.5307)^2 + (0.7105)^2 - 2(-0.5307)(0.7105)(-0.1271)\}^{\frac{1}{2}} = 0.8299$$

$$O_bO_c = \{(-0.2297)^2 + (-0.3411)^2 - 2(-0.2297)(-0.3411)(-0.1271)\}^{\frac{1}{2}} = 0.4348$$

$$M = BD = \{(0.7105)^2 + (0.8192)^2 - 2(0.7105)(0.8192)\,0.7071\}^{\frac{1}{2}} = 0.5940$$

$$N = CD = \{(-0.5307)^2 + (0.8192)^2 - 2(-0.5307)(0.8192)\,0.6115\}^{\frac{1}{2}} = 1.2201$$

The linkage size index is

$$UN = \frac{0.9402 + 0.1896 + 0.8299 + 0.4348 + \dfrac{0.5940 + 1.2201}{2}}{5} = 0.6603$$

Dividing each calculated dimension by the UN yields the linkage, shown in Fig. 20, that provides the stated straight-line output of 1.42 inches at an accuracy of 0.01 inch.

$$O_bB = \frac{0.9402}{0.6603} = 1.42'' = R$$

$$O_cC = \frac{-0.1896}{0.6603} = -0.287'' = T$$

$$BC = \frac{0.8299}{0.6603} = 1.26'' = S$$

$$O_bO_c = \frac{0.4348}{0.6603} = 0.658'' = Q$$

$$BD = \frac{0.5940}{0.6603} = 0.899'' = M$$

$$CD = \frac{1.2201}{0.6603} = 1.85'' = N$$

There are many additional linkages that satisfy the initial requirement that can be tried should above dimensions or output prove unsatisfactory.

PARAMETERS

$\alpha_a = 22.0$ $\alpha_b = 10.0$

DIMENSIONS ($\alpha_d = 55.000$)

Q =	0.6585	S =	1.2583
R =	1.4239	M =	0.8993
T =	0.2873	ϵ =	-116.5

RESULTS ($\phi_o = 51.079$)

D	0.0001	0.001	0.01
L	0.570	1.120	1.404
$\Delta\psi$	73.9	147.9	196.8
γ_c	37.3	23.9	4.6

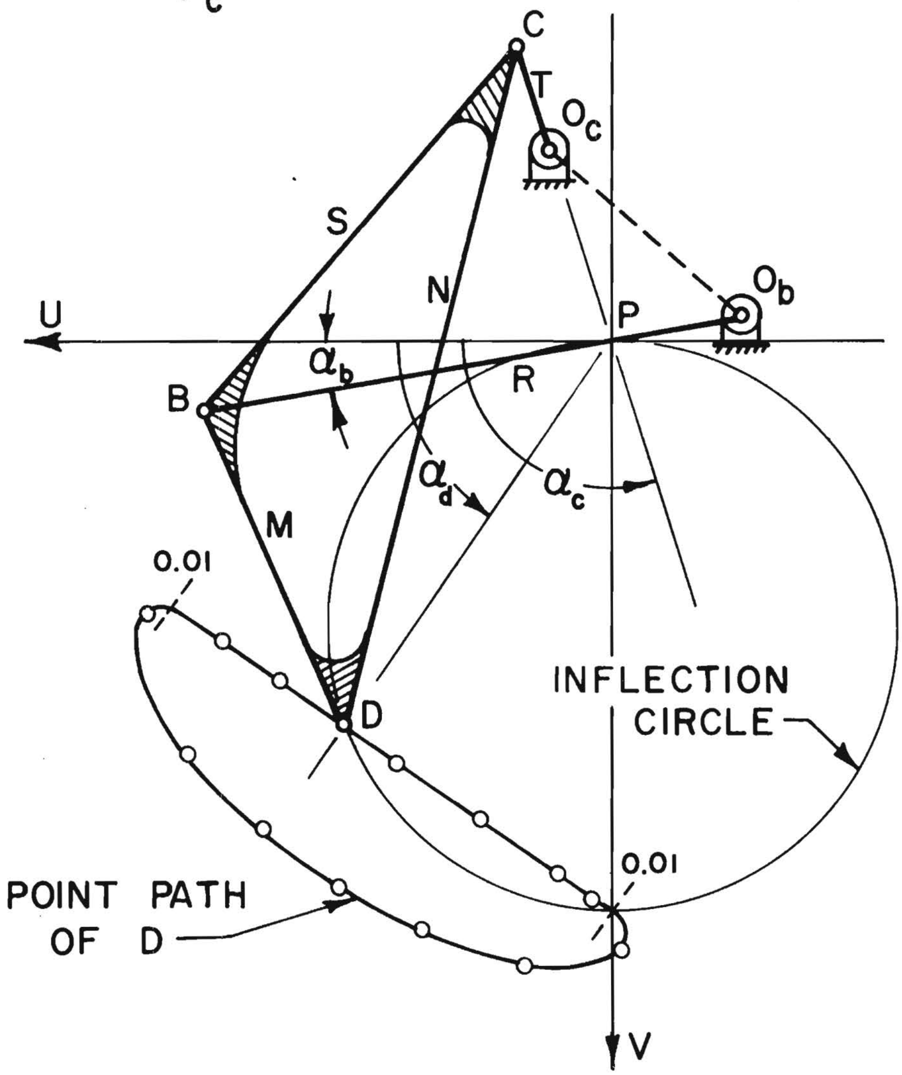
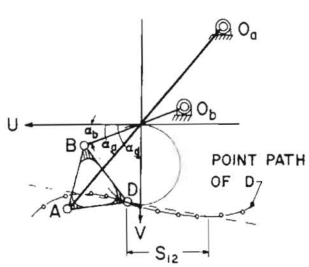
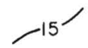


Figure 20. Ball-Burmester*-*Intersection Mechanism.

BALL—BURMESTER POINT WITH FINITE INTERSECTION





CONTOUR NUMBERS FOR ANGLE α_b WHICH TOGETHER WITH α_a AND α_d WILL GENERATE A MECHANISM HAVING AN INTERSECTION WITH EXACT STRAIGHT LINE AT DISTANCE S_{12}.

$S_{12} = 0.6$

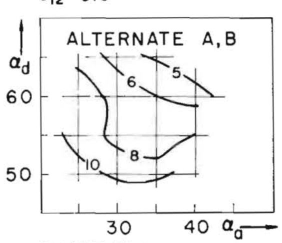


CHART FI 1

$S_{12} = 0.7$

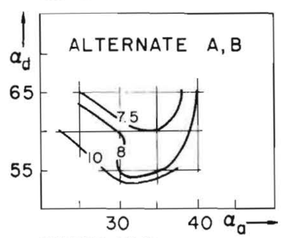


CHART FI 2

$S_{12} = 0.05$

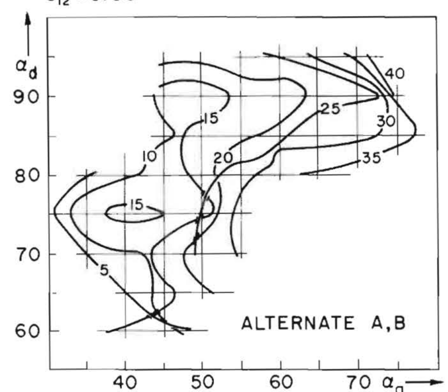


CHART FI 5

$S_{12} = 0.09$

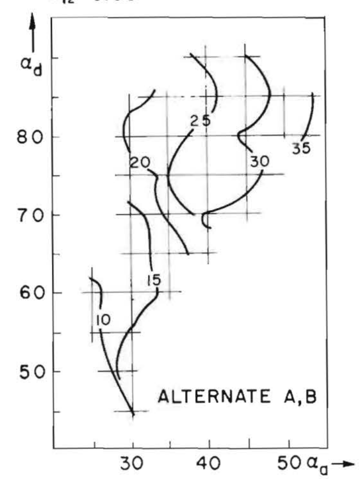


CHART FI 7

$S_{12} = 0.8$

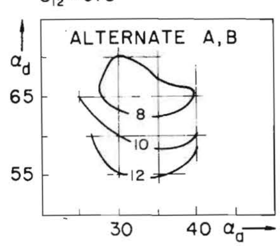


CHART FI 3

$S_{12} = 0.9$

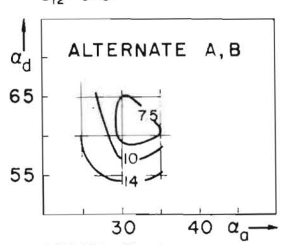


CHART FI 4

$S_{12} = 0.07$

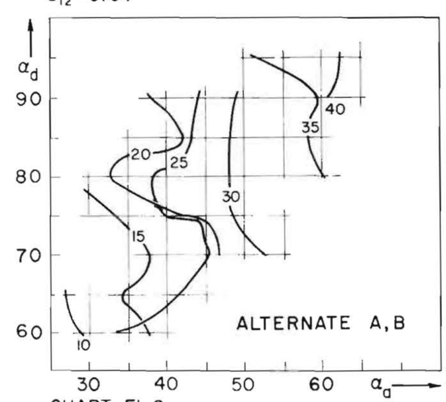
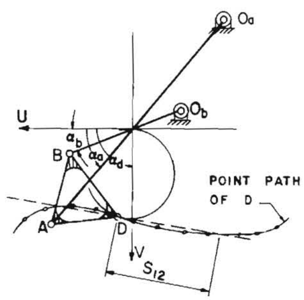


CHART FI 6

MEANS LINE a IS ABOVE LINE b

FI CHARTS TO BE USED IN CONJUNCTION WITH BB CHARTS

BALL-BURMESTER POINT WITH FINITE INTERSECTION



15 — CONTOUR NUMBERS FOR ANGLE α_b WHICH TOGETHER WITH α_a AND α_d WILL GENERATE A MECHANISM HAVING AN INTERSECTION WITH EXACT STRAIGHT LINE AT DISTANCE S_{12}

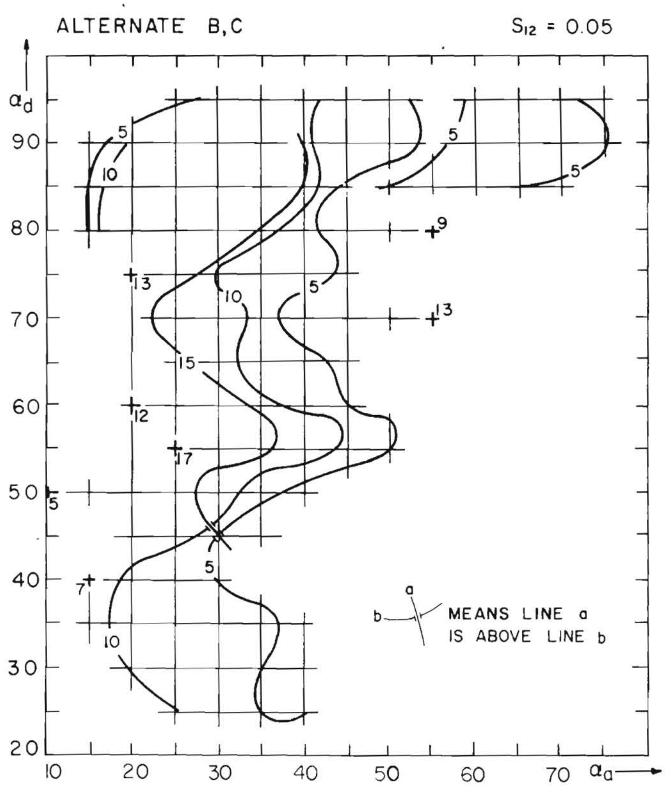


CHART FI 8A

FI CHARTS TO BE USED IN CONJUNCTION WITH BB CHARTS

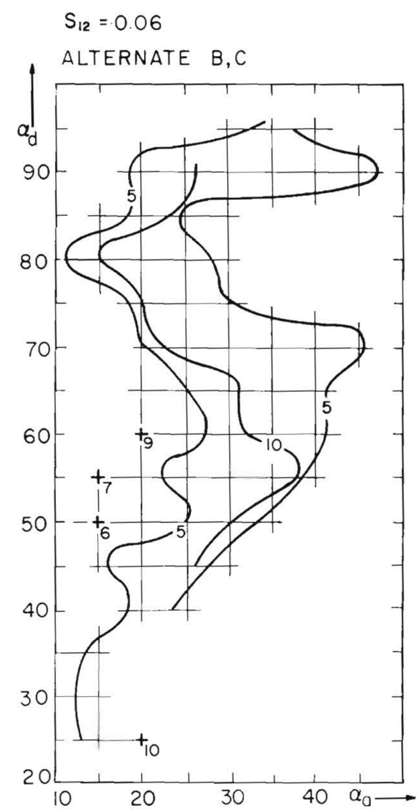
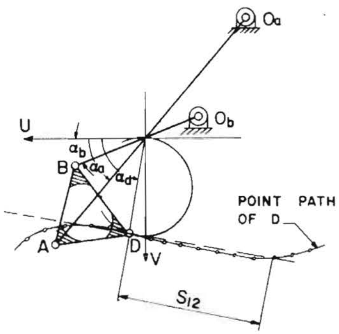
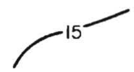


CHART FI 9

BALL—BURMESTER POINT WITH FINITE INTERSECTION





CONTOUR NUMBERS FOR ANGLE α_b WHICH TOGETHER WITH α_a AND α_d WILL GENERATE A MECHANISM HAVING AN INTERSECTION WITH EXACT STRAIGHT LINE AT DISTANCE S_{12}

ALTERNATE B,C

FI CHARTS TO BE USED IN CONJUNCTION WITH BB CHARTS

S_{12} = 0.05

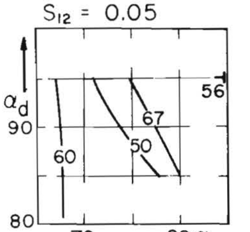
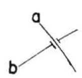


CHART FI 8B



MEANS LINE a IS ABOVE LINE b

S_{12} = 0.08

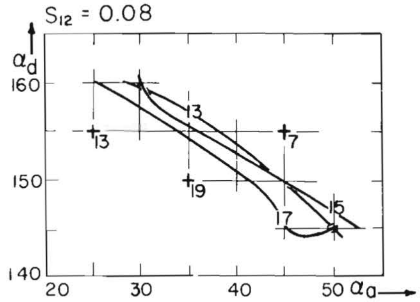


CHART FI 10

S_{12} = 0.4

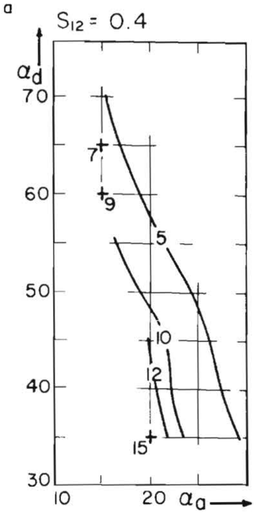


CHART FI 11

S_{12} = 0.6

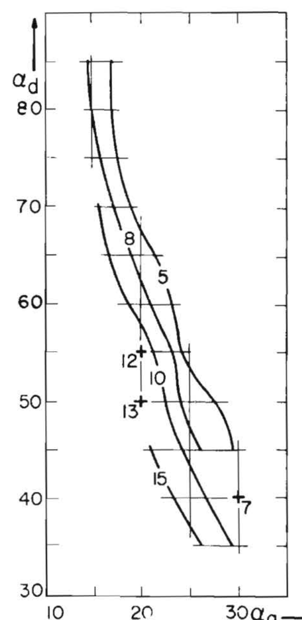


CHART FI 12

S_{12} = 0.8

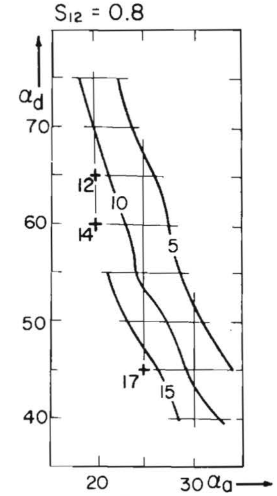


CHART FI 13